Tree Automata and Applications

Exercise session 1

Luc Lapointe
luc.lapointe@ens-paris-saclay.fr
home.lmf.cnrs.fr/LucLapointe/

Exercise 1 - First constructions of Tree Automata

Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down DFTA for the set $G(t)$ of ground instances of the term $t = f(f(a,x), g(y))$ which is defined by:

$$G(t) = \left\{ f(f(a,u), g(v)) \mid u, v \in T(\mathcal{F}) \right\}$$

Exercise 2 - What is recognizable by a tree automaton?

Are the following tree languages recognizable by a bottom-up tree automaton?

1. $\mathcal{F} = \{g(1), a(0)\}$ and L the set of ground terms of even height.
2. $\mathcal{F} = \{f(2), g(1), a(0)\}$ and L the set of ground terms of even height.

Exercise 3 - Bottom-up vs Top-down

1. Recall why bottom-up NFTAs, bottom-up DFTAs and top-down NFTAs have the same expressiveness.
2. Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down NFTA for the set $M(t)$ of terms which have a ground instance of the term $t = f(a, g(x))$ as a subterm, i.e.

$$\left\{ C\left[f(a, g(u))\right] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F}) \right\}.$$

3. Show that NFTAs and top-down DFTAs do not have the same expressiveness.

Homework: Satisfiability

Let $\mathcal{F} = \{\text{and}(2), \text{or}(2), \text{not}(1), 0(0), 1(0), x(0)\}$. A ground term over $\mathcal{F}$ can be viewed as a boolean formula over x.

1. Give an NFTA which recognizes the set of satisfiable boolean formulae over x. Show that it indeed does.

Let $\mathcal{F} = \left\{\text{and}(2), \text{or}(2), \text{not}(1), 0(0), 1(0), x_1(0), ..., x_{n(0)}\right\}$, i.e. we now handle n variables instead of a single one. The same variable may appear several times in a formula, and should be evaluated consistently.

2. Give an NFTA which recognizes the set of satisfiable boolean formulae over $x_1, ..., x_n$. Show that it does.