Tree Automata and Applications
Exercise session 2

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Exercise 1 - Quizz

Be precise in your answers.

1. What is the most expressive, top-down or bottom-up?
2. What are the general techniques to prove that an automaton recognizes a language L?
3. What is the minimal number of states required by a bottom-up NFTA to recognize the language made of a single term with one node and two leaves?
4. Can the language of trees such that all branches contain a specific symbol f be recognized by a NFTA?
5. Can it be recognized by a top-down DFTA?

Exercise 2 - Tree Automata

Let $\mathcal{F} = \{f(2), g(2), a(0)\}$. Give NFTAs for the following languages:

1. $\{t \in \mathcal{F} \mid \text{on some branch in } t \text{ there are two consecutive occurrences of } f\}$
2. $\{t \in \mathcal{F} \mid \text{on all branches in } t \text{ there are two consecutive occurrences of } f\}$

Exercise 3 - Regular expressions

1. Describe the language recognized by the following regular expressions over alphabet $\mathcal{F} = \{f(2), g(2), a(0), b(0)\}$:

$$E = f(\Box_1, \Box_1)^{*\Box_1} \cdot \Box_1 \left[g(\Box_2, \Box_2) \cdot \Box_2 (f(\Box_1, \Box_1))^{*\Box_1} \cdot \Box_1 b \right]$$

2. Give a regular expression for:
   1. the set $T(\mathcal{F})$ of all finite trees on alphabet $\mathcal{F} = \{f(2), g(2), a(0), b(0)\}$;
   2. $\{t \in T(\mathcal{F}) \mid t \text{ contains the subtree } f(a, b)\}$ where $\mathcal{F} = \{f(2), a(0), b(0)\}$;
   3. $\{t \in T(\mathcal{F}) \mid \text{the frontier word of } t \text{ contains an infix } ab\}$ with same $\mathcal{F}$.

Homework - Satisfiability (again)

Let $\mathcal{F} = \{\text{and}(2), \text{or}(2), \text{not}(1), 0(0), 1(0), x(1), s(1), z(0)\}$, i.e. we now handle an arbitrary number of variables instead of a fixed one (encoding x_2 as $x(s(s(z)))$). The same variable may appear several times in a formula, and should be evaluated consistently.

1. Is the set of well-formed formulae using this syntax recognizable by an NFTA?
2. Is the set of satisfiable formulae using this syntax recognizable by an NFTA?