Tree Automata and Applications
Exercise session 4

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Exercise 1 - WS2S on finite trees

We consider trees with maximum arity 2. Give WS2S formulae which express the following:

1. $x \subseteq y$, with $\subseteq$ the prefix relation on positions.
2. X is closed under predecessors.
3. The letter a occurs twice on the same path.
4. The letter a occurs twice not on the same path.
5. There exists a subtree with only a letters.

Exercise 2 - The limit of WSkS

Prove that the predicate $x = 1y$ is not definable in WSkS.

Exercise 3 - The power of WSkS

Produce formulae of WSkS for the following predicates:

1. The set X has exactly two elements.
2. The set X contains at least one string beginning with a 1.
3. Given a formulae of WSkS φ with one free first-order variable, produce a formula of WSkS expressing that there is an infinity of words on $\{1, \ldots, k\}^*$ satisfying φ.

Homework - From formulae to automata

Give tree automata recognizing the languages on trees of maximum arity 2 defined by the following formulae:

1. $(x \in X \wedge (x1 = y \Rightarrow y \in X)) \wedge \big(z \in X \Rightarrow P_f(z)\big)$
2. $\exists X \big[(x \in X \wedge (x1 = y \Rightarrow y \in X)) \wedge \big(z \in X \Rightarrow P_f(z)\big)\big]$