Cheatsheet

Nicolas Dumange nicolas.dumange@ens-paris-saclay.fr

Définition —— Déduction naturelle

Règles de base ($*: y$ non libre dans $\Gamma \cup \{\varphi, \psi\}$)

$$\frac{}{\Gamma, \varphi \vdash \varphi}\text{Ax} \qquad \frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1, \Gamma_2 \vdash \varphi \wedge \psi}\wedge_I \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi}\wedge_{E_L} \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi}\wedge_{E_R}$$

$$\frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}\bot_E \qquad \frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi}\to_I \qquad \frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \varphi \to \psi}{\Gamma_1, \Gamma_2 \vdash \psi}\to_E \qquad \frac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \varphi}C$$

$$\frac{\Gamma \vdash \forall x.\varphi}{\Gamma \vdash \varphi[t/x]}\forall_E \qquad \frac{\Gamma \vdash \varphi[y/x]}{\Gamma \vdash \forall x.\varphi}\forall_I * \qquad \frac{\Gamma \vdash \varphi[t/x]}{\Gamma \vdash \exists x.\varphi}\exists_{I_R} \qquad \frac{\Gamma, \varphi[y/x] \vdash \psi}{\Gamma, \exists x.\varphi \vdash \psi}\exists_{I_L} *$$

Règles admissibles vues en TD:

$$\frac{\Gamma \vdash \varphi_1}{\Gamma \vdash \varphi_1 \vee \varphi_2}\vee_{I_L} \qquad \frac{\Gamma \vdash \varphi_2}{\Gamma \vdash \varphi_1 \vee \varphi_2}\vee_{I_R} \qquad \frac{\Gamma, \varphi \vdash \bot}{\Gamma \vdash \neg\varphi}\neg_I \qquad \frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \neg\varphi}{\Gamma_1 \cup \Gamma_2 \vdash \bot}\neg_E$$

$$\frac{\Gamma_0 \vdash \varphi_1 \vee \varphi_2 \qquad \Gamma_1, \varphi_1 \vdash \psi \qquad \Gamma_2, \varphi_2 \vdash \psi}{\Gamma_0 \cup \Gamma_1 \cup \Gamma_2 \vdash \psi}\vee_E \qquad \frac{\Gamma, \varphi \vdash \psi \qquad \Gamma \vdash \varphi}{\Gamma \vdash \psi}\text{Cut}$$

Règles équivalentes à C (non constructives):

$$\frac{}{\Gamma \vdash \varphi \vee \neg\varphi}\text{EM} \qquad \frac{\Gamma \vdash \neg\neg\varphi}{\Gamma \vdash \varphi}\text{Abs}$$

Définition —— Calcul des séquents

LK : ($*$: x non libre dans $\Gamma \cup \Delta$)

$$\frac{\Gamma, \varphi, \psi, \Gamma' \vdash \Delta}{\Gamma, \psi, \varphi, \Gamma' \vdash \Delta}\,\mathcal{L}X \qquad \frac{\Gamma \vdash \Delta, \varphi, \psi, \Delta'}{\Gamma \vdash \Delta, \psi, \varphi, \Delta'}\,\mathcal{R}X \qquad \frac{\Gamma \vdash \Delta}{\Gamma, \varphi \vdash \Delta}\,\mathcal{L}W \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \varphi}\,\mathcal{R}W$$

$$\frac{\Gamma, \varphi, \varphi \vdash \Delta}{\Gamma, \varphi \vdash \Delta}\,\mathcal{L}C \qquad \frac{\Gamma \vdash \Delta, \varphi, \varphi}{\Gamma \vdash \Delta, \varphi}\,\mathcal{R}C \qquad \frac{}{\varphi \vdash \varphi}\,\mathrm{Ax} \qquad \frac{\Gamma \vdash \varphi, \Delta \qquad \Gamma', \varphi \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,\mathrm{Cut}$$

$$\frac{\Gamma, \varphi \vdash \Delta}{\Gamma, \varphi \wedge \psi \vdash \Delta}\,\mathcal{L}\wedge_L \qquad \frac{\Gamma, \psi \vdash \Delta}{\Gamma, \varphi \wedge \psi \vdash \Delta}\,\mathcal{L}\wedge_R \qquad \frac{\Gamma \vdash \varphi, \Delta \qquad \Gamma' \vdash \psi, \Delta'}{\Gamma, \Gamma' \vdash \varphi \wedge \psi, \Delta, \Delta'}\,\mathcal{R}\wedge$$

$$\frac{\Gamma, \varphi \vdash \Delta \qquad \Gamma', \psi \vdash \Delta'}{\Gamma, \Gamma', \varphi \vee \psi \vdash \Delta, \Delta'}\,\mathcal{L}\vee \qquad \frac{\Gamma \vdash \varphi, \Delta}{\Gamma \vdash \varphi \vee \psi, \Delta}\,\mathcal{R}\vee_L \qquad \frac{\Gamma \vdash \psi, \Delta}{\Gamma \vdash \varphi \vee \psi, \Delta}\,\mathcal{R}\vee_R$$

$$\frac{\Gamma \vdash \varphi, \Delta \qquad \Gamma', \psi \vdash \Delta'}{\Gamma, \Gamma', \varphi \to \psi \vdash \Delta, \Delta'}\,\mathcal{L}\to \qquad \frac{\Gamma, \varphi \vdash \psi, \Delta}{\Gamma \vdash \varphi \to \psi, \Delta}\,\mathcal{R}\to$$

$$\frac{\Gamma, \varphi[y/x] \vdash \Delta}{\Gamma, \forall x.\varphi \vdash \Delta}\,\mathcal{L}\forall \qquad \frac{\Gamma \vdash \varphi, \Delta}{\Gamma \vdash \forall x.\varphi, \Delta}\,\mathcal{R}\forall * \qquad \frac{\Gamma, \varphi \vdash \Delta}{\Gamma, \exists x.\varphi \vdash \Delta}\,\mathcal{L}\exists * \qquad \frac{\Gamma \vdash \varphi[t/x], \Delta}{\Gamma \vdash \exists x.\varphi, \Delta}\,\mathcal{R}\exists$$

LJ : ($*$: x non libre dans $\Gamma \cup \{\delta\}$)

$$\frac{\Gamma, \varphi, \psi, \Gamma' \vdash \delta}{\Gamma, \psi, \varphi, \Gamma' \vdash \delta}\,\mathcal{L}X \qquad \frac{\Gamma \vdash \delta}{\Gamma, \varphi \vdash \delta}\,\mathcal{L}W \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}\,\mathcal{R}W \qquad \frac{\Gamma, \varphi, \varphi \vdash \delta}{\Gamma, \varphi \vdash \delta}\,\mathcal{L}C$$

$$\frac{}{\varphi \vdash \varphi}\,\mathrm{Ax} \qquad \frac{\Gamma \vdash \varphi \qquad \Gamma', \varphi \vdash \delta}{\Gamma, \Gamma' \vdash \delta}\,\mathrm{Cut}$$

$$\frac{\Gamma, \varphi \vdash \delta}{\Gamma, \varphi \wedge \psi \vdash \delta}\,\mathcal{L}\wedge_L \qquad \frac{\Gamma, \psi \vdash \delta}{\Gamma, \varphi \wedge \psi \vdash \delta}\,\mathcal{L}\wedge_R \qquad \frac{\Gamma \vdash \varphi \qquad \Gamma' \vdash \psi}{\Gamma, \Gamma' \vdash \varphi \wedge \psi}\,\mathcal{R}\wedge$$

$$\frac{\Gamma, \varphi \vdash \delta \qquad \Gamma', \psi \vdash \delta}{\Gamma, \Gamma', \varphi \vee \psi \vdash \delta}\,\mathcal{L}\vee \qquad \frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi}\,\mathcal{R}\vee_L \qquad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi}\,\mathcal{R}\vee_R$$

$$\frac{\Gamma \vdash \varphi \qquad \Gamma', \psi \vdash \delta}{\Gamma, \Gamma', \varphi \to \psi \vdash \delta}\,\mathcal{L}\to \qquad \frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi}\,\mathcal{R}\to$$

$$\frac{\Gamma, \varphi[y/x] \vdash \delta}{\Gamma, \forall x.\varphi \vdash \delta}\,\mathcal{L}\forall \qquad \frac{\Gamma \vdash \varphi}{\Gamma \vdash \forall x.\varphi}\,\mathcal{R}\forall * \qquad \frac{\Gamma, \varphi \vdash \delta}{\Gamma, \exists x.\varphi \vdash \delta}\,\mathcal{L}\exists * \qquad \frac{\Gamma \vdash \varphi[t/x]}{\Gamma \vdash \exists x.\varphi}\,\mathcal{R}\exists$$