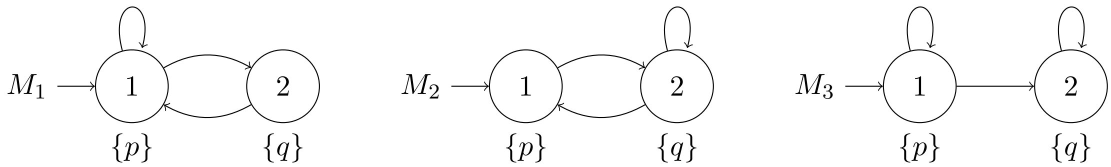
Exercice session 4: Büchi automata, CTL

Exercise 1 (CTL)**.** For each model and each formula, say if the model satisfies the formula.



1. $\phi_1 = \mathsf{AGAF}q$
2. $\phi_2 = \mathsf{EGEF}q$
3. $\phi_3 = \mathsf{AFEG}p$
4. $\phi_4 = \mathsf{AGEF}q$

Exercise 2 (CTL Equivalences)**.**

1. Are the two formulæ $\varphi = \mathsf{A}\,\mathsf{G}(\mathsf{E}\,\mathsf{F}\,p)$ and $\psi = \mathsf{E}\,\mathsf{F}\,p$ equivalent? Does one imply the other?
2. Same questions for $\varphi = \mathsf{E}\,\mathsf{G}\,q \vee (\mathsf{E}\,\mathsf{G}\,p \wedge \mathsf{E}\,\mathsf{F}\,q)$ and $\psi = \mathsf{E}(p\,\mathsf{U}\,q)$.

Exercise 3 (Büchi Automata)**.** For each formula, build the Büchi automaton using the method seen during the course.

1. $a\,\mathsf{U}\,(\mathsf{X}\,b \wedge \neg a)$
2. $a \wedge \mathsf{G}(a \to \mathsf{X}\,b) \wedge \mathsf{G}(b \to \mathsf{X}\,a)$
3. $\mathsf{G}(a \to (b\,\mathsf{U}\,c))$
4. $\mathsf{G}\,\mathsf{F}\,b \to \mathsf{F}\,\mathsf{G}\,a$

Exercise 4 (CTL$^+$)**.** CTL$^+$ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

$$f ::= \top \mid a \mid f \wedge f \mid \neg f \mid \mathsf{E}\,\varphi \mid \mathsf{A}\,\varphi \qquad \text{(state formulæ)}$$
$$\varphi ::= \varphi \wedge \varphi \mid \neg\varphi \mid \mathsf{X}\,f \mid f\,\mathsf{U}\,f \qquad \text{(path formulæ)}$$

where a is an atomic proposition. The associated semantics is

$$\begin{array}{ll} \mu, s \models \mathsf{E}\,\varphi & \text{iff } \pi \models \varphi \text{ for some path } \pi \text{ starting with } s \\ \mu, s \models \mathsf{A}\,\varphi & \text{iff } \pi \models \varphi \text{ for all path } \pi \text{ starting with } s \\ \pi \models \mathsf{X}\,f & \text{iff } \mu, \pi[1] \models f \\ \pi \models f \,\mathsf{U}\, g & \text{iff } \exists k \geq 0 : \mu, \pi[k] \models g \text{ and } \forall j, 0 \leq j < k : \pi[j] \models f \end{array}$$

with the semantics for (path and state) boolean operators being the one expected.

We want to prove that, for any CTL$^+$ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for

$$\mathsf{E}((a_1 \,\mathsf{U}\, b_1) \wedge (a_2 \,\mathsf{U}\, b_2)) \ .$$

2. Generalize your translation for any formula of form

$$\mathsf{E}\left(\bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi_i') \wedge \mathsf{G}\,\varphi\right) \ . \tag{1}$$

   What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL$^+$ formula:

$$\mathsf{E}(\mathsf{X}\,a \wedge (b \,\mathsf{U}\, c)) \ .$$

4. Using subformulæ of form (1) and E modalities, give an equivalent CTL formula to

$$\mathsf{E}(\mathsf{X}\,\varphi \wedge \bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi_i') \wedge \mathsf{G}\,\varphi') \ . \tag{2}$$

   What is the complexity of your translation?

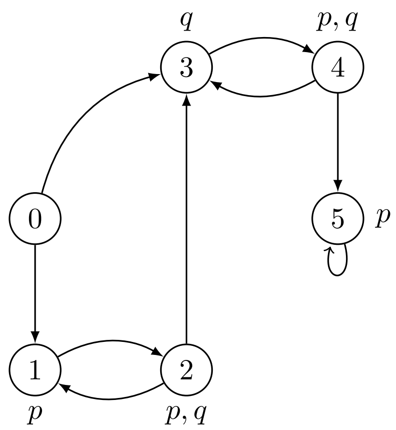
5. We only have to transform any CTL$^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:

$$\mathsf{A}((\mathsf{F}\,a \vee \mathsf{X}\,a \vee \mathsf{X}\,\neg b \vee \mathsf{F}\,\neg d) \wedge (d \,\mathsf{U}\, \neg c)) \ .$$

Homework

To hand in on October 25 or anytime sooner by email at `nicolas.dumange@ens-paris-saclay.fr`, with file name of the form "FirstnameLastname.pdf".
Answers can be written in french or in english.

We give the following Kripke structure:



and the following CTL formula:

$$\phi = \mathsf{E}((\mathsf{AF}\,\mathsf{EG}\,p)\ \mathsf{U}\ (\mathsf{AG}\,\mathsf{EX}\,q)) \vee \mathsf{A}((\mathsf{EX}\,\mathsf{EG}\,p)\ \mathsf{U}\ q)$$

For each state sub-formula ψ of ϕ, compute the set of states satisfying ψ.