Exercice session 5: Bisimulation, CTL

Exercise 1 (Bisimulation game)**.** Consider the game $\mathcal{G}(\mathcal{K},\mathcal{K}')$ with $\mathcal{K}$ and $\mathcal{K}'$ being Kripke structures labeled over the same set AP of atomic proposition. In this game, the *Challenger*, or P_0, and the *Verifier*, or P_1, move two pebbles along the states of the Kripke stuctures.

We call a *configuration* the pair (s, s') of states with s being a state in $\mathcal{K}$ et s' a state in $\mathcal{K}'$, denoting the position of the pebbles. A configuration (s, s') is said *compatible* iff the labeling of s and s' are the same.
On each turn, the Challenger chooses either a state r to move the pebble onto such that $s \to r$ is a transition in $\mathcal{K}$ or r' such that $s' \to r'$ is a transition in $\mathcal{K}'$, and the Verifier has to respond by movining the other pebble onto a successor state such that the new configuration is compatible.

The Challenger wins the game $\mathcal{G}(\mathcal{K},\mathcal{K}')$ if the Verifier can not play anymore, and the Verifier wins otherwise.

Admitting that the game is determined (either the Challenger or the Verifier has a winning strategy), show that $\mathcal{K}$ and $\mathcal{K}'$ are bisimilar iff the Verifier has a winning strategy.

Exercise 2 (2017 Mid-term Exam)**.** The flow of time is $(\mathbb{N}, <)$, AP is the set of atomic propositions, and $\Sigma = 2^{\mathrm{AP}}$. We denote Σ_p the subset of Σ whose elements contain p, and $\Sigma_{\neg p}$ whose elements do not contain p.

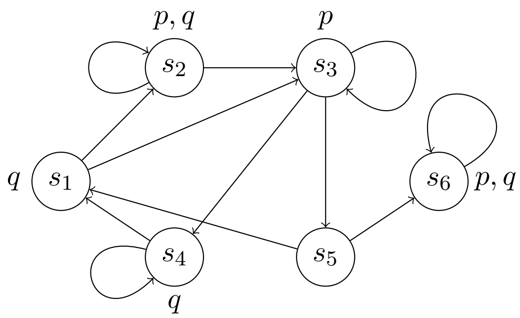
1. Given $p \in \mathrm{AP}$ and $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$, construct a formula $\widetilde{\varphi} \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ such that

$$\forall u \in \Sigma_{\neg p}^* \Sigma_p,\ \forall v \in \Sigma^\omega,\ \forall i \geq 0 : \qquad v, i \models \varphi \quad \text{iff} \quad uv, |u| + i \models \widetilde{\varphi}\,.$$

2. Given $p \in \mathrm{AP}$ and $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$, construct a formula $\overline{\varphi} \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ such that

$$\forall u \in \Sigma_{\neg p}^* \Sigma_p,\ \forall v \in \Sigma^\omega : \qquad v, 0 \models \varphi \quad \text{iff} \quad uv, 0 \models \overline{\varphi}\,.$$

Exercise 3 (CTL revision)**.** Consider the following Kripke structure $\mathcal{K}$



Check whether the following statements hold.

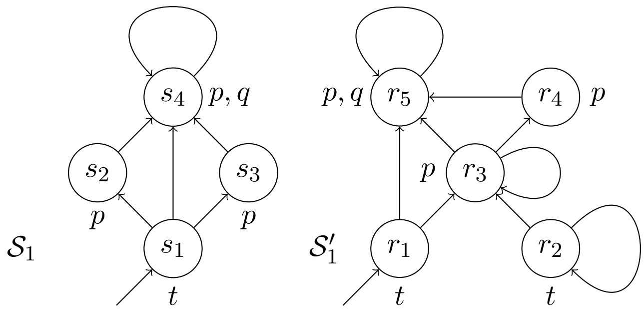
1. $\mathcal{K}, s_2 \models \mathsf{EG}\,\mathsf{EF}\,p$
2. $\mathcal{K}, s_3 \models \mathsf{AF}\,\mathsf{EG}(p \vee q)$
3. $\mathcal{K}, s_1 \models \mathsf{AF}(q \wedge \mathsf{EX}\,p)$
4. $\mathcal{K}, s_3 \models \mathsf{EG}\,\mathsf{EF}(\mathsf{E}(\mathsf{EF}\,p)\,\mathsf{U}\,q)$
5. $\mathcal{K}, s_1 \models \mathsf{AG}\,\mathsf{EF}\,q \to \mathsf{EG}\,\mathsf{EF}\,q$
6. $\mathcal{K}, s_4 \models (q \wedge \mathsf{AX}\,\mathsf{E}(q\,\mathsf{U}\,p)) \to \mathsf{E}(q\,\mathsf{U}\,p)$

Homework

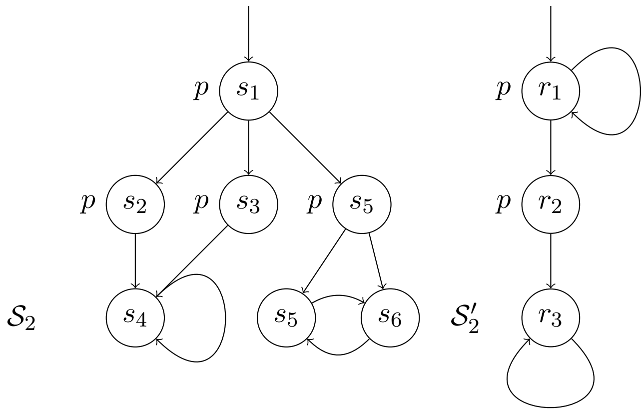
To hand in on November 7 or anytime sooner by email at `nicolas.dumange@ens-paris-saclay.fr`, with file name of the form "FirstnameLastname.pdf".
Answers can be written in french or in english.

- Determine for each pair of Kripke structure, whether they are bisimilar or not. If so give a bisimulation, if not give a CTL formula distinguishing them.

1.



2.



- Same question for each pair the three following Kripke structures.