Local testing of MSCs

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31 January 2008

Plan

1. MSC

HMSC

Local testing of HMSC

Undecidability of 1-testability for 2 processes

Undecidability of k-testability

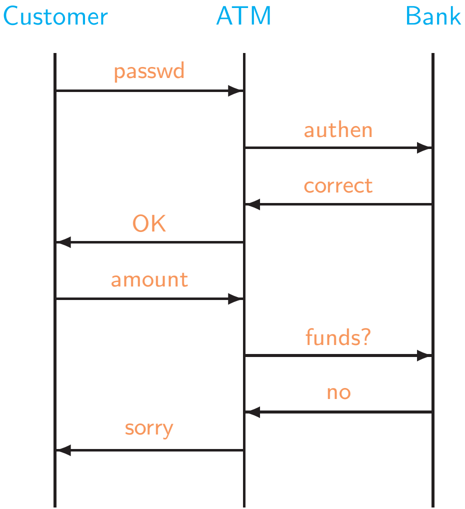
Decidability of 1-testability without message contents

Scenarios

- A scenario describes a pattern of interaction
- Attractive visual formalism
- Telecommunications
  - Message sequence charts (MSC)
  - Messages sent between communicating agents
- UML
  - Sequence diagrams
  - Interaction between objects

    e.g., method invocations etc

An ATM


Customer
ATM
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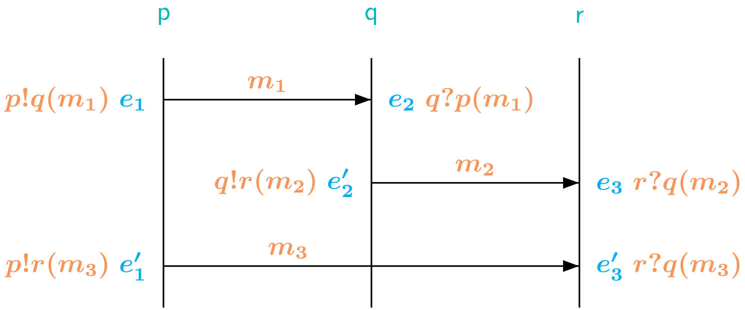
sorry

MSCs as labelled partial orders



- Linearizations give a word language
  $p!q(m_1)\ p!r(m_3)\ q?p(m_1)\ q!r(m_2)\ r?q(m_2)\ r?q(m_3)$,
  $p!q(m_1)\ q?p(m_1)\ q!r(m_2)\ p!r(m_3)\ r?q(m_2)\ r?q(m_3)$,...
- A single linearization is sufficient to reconstruct MSC

Plan

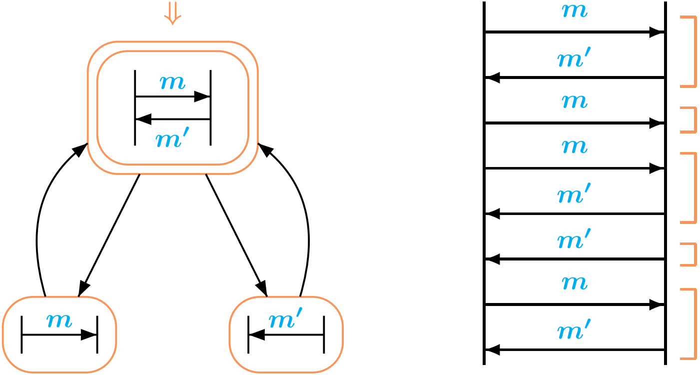
1. MSC
2. HMSC
3. Local testing of HMSC
4. Undecidability of 1-testability for 2 processes
5. Undecidability of k-testability
6. Decidability of 1-testability without message contents

Collections of MSCs

- Often need to specify a collection of scenarios
- Finite collection can be exhaustively enumerated
- Infinite collection needs a generating mechanism

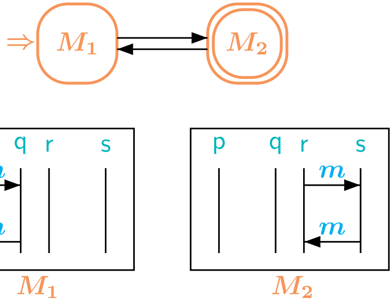
High level MSCs (HMSCs)

- A finite state automaton
- Each state is labelled by an MSC
- Each (legal) path in the automaton generates an MSC



HMSC semantics

- All processes must traverse the same path in an HMSC
- ... but processes move asynchronously
- Some processes may be (unboundedly) far ahead of others



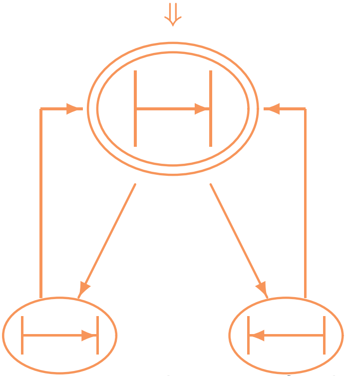
- After k iterations, we could have r and s in the final copy of M_2 while p and q are in the first copy of M_1

Regular MSC languages

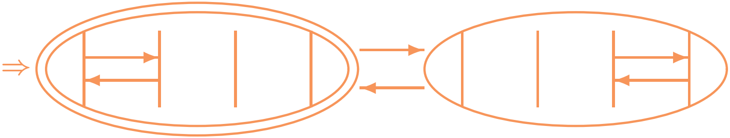
- An MSC is (uniquely) determined by its linearizations
  - Set of strings over send actions $p!q(m)$ and receive actions $p?q(m)$
- Collection of MSCs $\Leftrightarrow$
  word language over send/receive actions
- Regular collection of MSCs $\triangleq$
  linearizations form a regular language

HMSCs and regularity

- HMSC specifications may not be regular
- Problem 1 Unbounded buffers



- Problem 2 Global synchronization yields context-free behaviours



HMSCs and regularity

- HMSC specifications may not be regular
- Problem 1 Unbounded buffers
- Problem 2 Global synchronization yields context-free behaviours
- Sufficient structural conditions on HMSCs to guarantee regularity ... [AY99,MP99]
  - Locally synchronized
- ... but checking if an HMSC specification is regular is undecidable [HMNST05]
- Every regular MSC language can be implemented as network of communicating finite-state automata with bounded channels [HMNST05]

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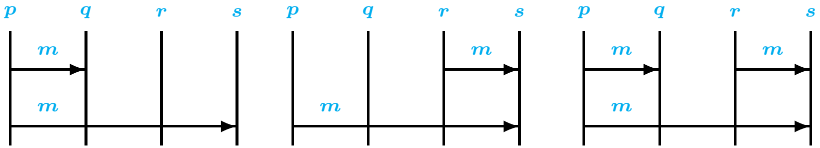
(Local) testing using scenarios

- Does an implementation conform to an HMSC specification?
- Local testing
  - Inject messages from some process(es) and observe the response
  - For each process p, local observer records sequence of events at p
  - If each local observation is consistent with some MSC defined by the HMSC, the implementation passes the test
- Does local testing suffice to check conformance of (regular) HMSC languages?

Implied scenarios [AEY00]

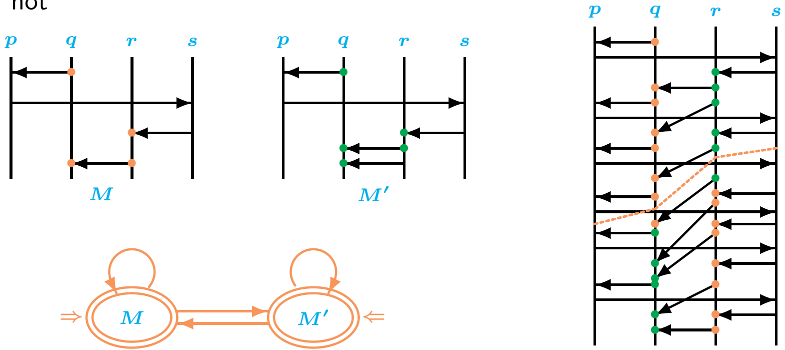


M_1 M_2 M

- p and q believe M is M_1
- r and s believe M is M_2
- MSC M is implied by L if for each process p, the p-projection of M matches the p-projection of some MSC in L
- An MSC language is locally testable if it is closed with respect to implied MSCs
- Originally studied in context of realizability

Implied scenarios . . .

- Even for regular MSC languages, checking local testability is undecidable! [AEY01]
- Even if the original language has bounded channels, its implied scenarios may not



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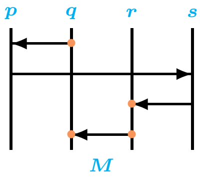
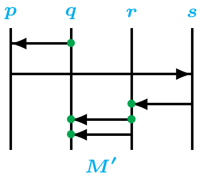
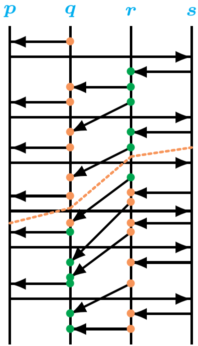


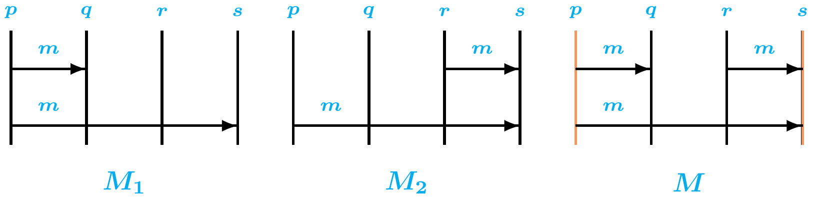




Confusing $M^{2k}M'^k$ and M'^kM^{2k} generates upto k messages in $p \longrightarrow s$ channel

Joint observations

- What if we have observers who can record the behaviours of sets of processes?



- M is detected as an illegal MSC by $\{p, s\}$.
- Joint observers have more discriminating power.

Joint observations . . .

- Fix some *observers* $P_1, P_2, \ldots, P_r$
- Each observer records the events on the processes in the set P_i

Given a HMSC G, is its language testable with observers $P_1, P_2, \ldots, P_r$?

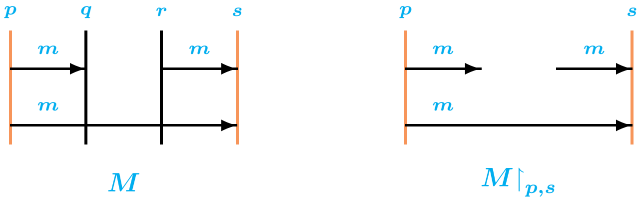
P-Observations

M an MSC, P a set of processes

P-observation of $M \triangleq$ tuple of projections of M on each process in P

$M{\restriction}_P$: P-observation of M.

$L{\restriction}_P = \{M{\restriction}_P \mid M \in L\}$: P-observation of a language L

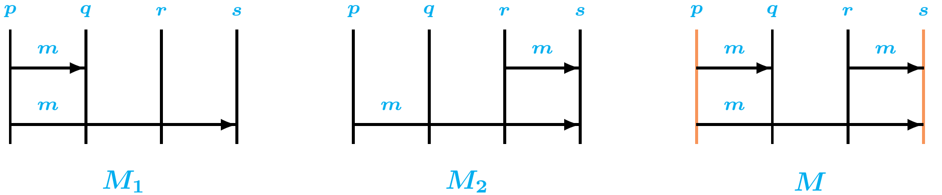


$$M{\restriction}_{p,s} = \langle p!q(m)p!s(m), s?r(m)s?p(m)\rangle.$$

k-testability

- Record P-observations for every set P of processes of size k.
- k-closure of a language $L \triangleq \{M \mid \forall P \text{ s.t. } |P| = k,\ M{\restriction_P} \in L{\restriction_P}\}$
- Scenario k-implied by $L \triangleq$ MSC in the k-closure of L but not in L
- A language is k-testable if it equals its k-closure
- Local testability is 1-testability

k-testability ...



The set $\{M_1, M_2\}$ is 2-testable but not 1-testable.

k-testability . . .

- 1-testability is undecidable for 4 or more processes. [AEY 01]
- n-testability is trivial
- What about k-testability for $1 < k < n$?
- What is the smallest $k \leq n$ such that k-testability is decidable?

Our results

- For all n and $k < n$ there are regular HMSC languages over n processes that are not k-testable
- k-testability is undecidable for $n \geq 3$ processes and $1 < k < n$
- 1-testability is undecidable for 2 processes
  - Improves result from 4 processes in [AEY01]
- k-testability remains undecidable for $n \geq 3$ processes and $1 < k < n$ even without message contents
- 1-testability is decidable without message contents

Plan

MSC

HMSC

Local testing of HMSC

4 Undecidability of 1-testability for 2 processes

Undecidability of k-testability

Decidability of 1-testability without message contents

1-testability for 2 processes

Theorem : 2 processes

For $n \geq 2$, 1-testability is undecidable for regular 4-bounded MSG-definable languages over n processes.

Proof :

Reduction to Modified Post Correspondence Problem (MPCP).

Modified Post Correspondence Problem

Definition : MPCP

Instance: sequence $(v_1, w_1), (v_2, w_2), \ldots, (v_r, w_r)$ of pairs of words such that

- $1 \leq |v_i| \leq 4$ and $1 \leq |w_i| \leq 4$ for $1 \leq i \leq r$,
- $w_1 < v_1$ and is shorter by at least 2 letters.

Solution: sequence $1 = i_1, i_2, i_3, \ldots, i_m$ of indices from $\{1, 2, \ldots, r\}$ such that

$$w_{i_1} w_{i_2} \cdots w_{i_m} = v_{i_1} v_{i_2} \cdots v_{i_m}$$

and for $k < m$,

$$w_{i_1} w_{i_2} \cdots w_{i_k} < v_{i_1} v_{i_2} \cdots v_{i_k}$$

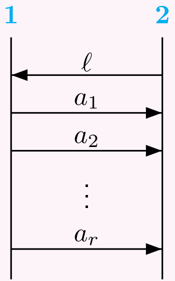
Theorem : MPCP

The Modified Post Correspondence Problem is undecidable.

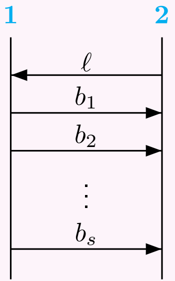
Undecidability: Reduction

Proof :

Let $\Delta = (v_1, w_1), (v_2, w_2), \ldots, (v_t, w_t)\}$ be an instance of the MPCP.



M_{v_ℓ} with $v_\ell = a_1 a_2 \ldots a_r$



M_{w_ℓ} with $w_\ell = b_1 b_2 \ldots b_s$

$$L_\Delta = M_{v_1}.\{M_{v_1}, \ldots, M_{v_t}\}^* + M_{w_1}.\{M_{w_1}, \ldots, M_{w_t}\}^*$$

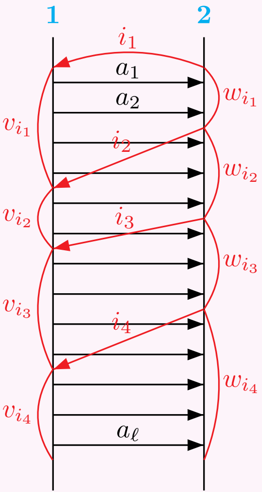
Lemma :

The MPCP Δ has a solution iff L_Δ has some 1-implied scenario.

Undecidability: Reduction

Proof : Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1} v_{i_2} \ldots v_{i_m} = a_1 a_2 \ldots a_\ell = w_{i_1} w_{i_2} \ldots w_{i_m}$$



$$L = M_{v_1}.\{M_{v_1}, \ldots, M_{v_t}\}^* + M_{w_1}.\{M_{w_1}, \ldots, M_{w_t}\}^*$$

- $M \notin L$
- $M\restriction_1 = (M_{v_{i_1}} M_{v_{i_2}} \ldots M_{v_{i_m}})\restriction_1 \in L\restriction_1$
- $M\restriction_2 = (M_{w_{i_1}} M_{w_{i_2}} \ldots M_{w_{i_m}})\restriction_2 \in L\restriction_2$

Plan

k-testability

Theorem : k-testability

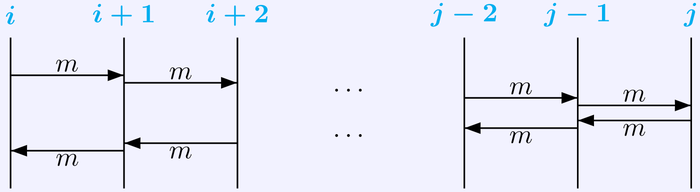
For $1 < k < n$, k-testability is undecidable for regular 1-bounded MSG-definable languages over n processes.

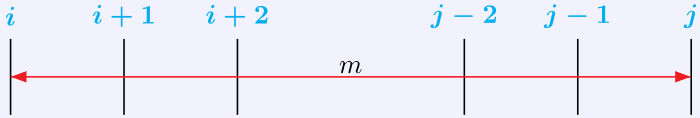
Proof :

Reduction to Modified Post Correspondence Problem (MPCP).

Undecidability: Reduction

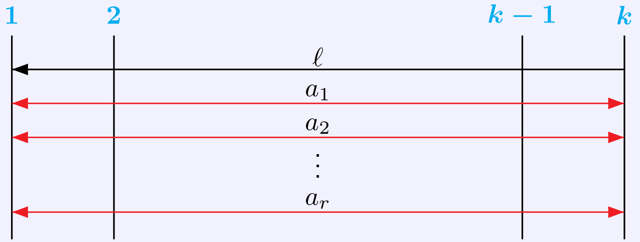
A basic MSC



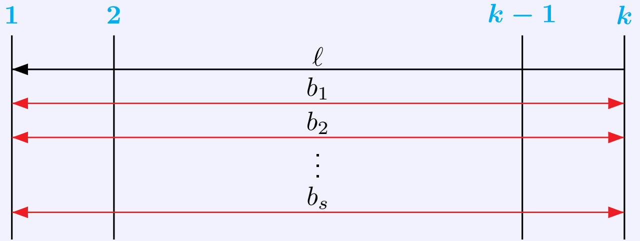


Undecidability: Reduction

Let $\Delta = (v_1, w_1), (v_2, w_2), \ldots, (v_t, w_t)\}$ be an instance of the MPCP.



M_{v_ℓ} with
$v_\ell = a_1 a_2 \ldots a_r$



M_{w_ℓ} with
$w_\ell = b_1 b_2 \ldots b_s$

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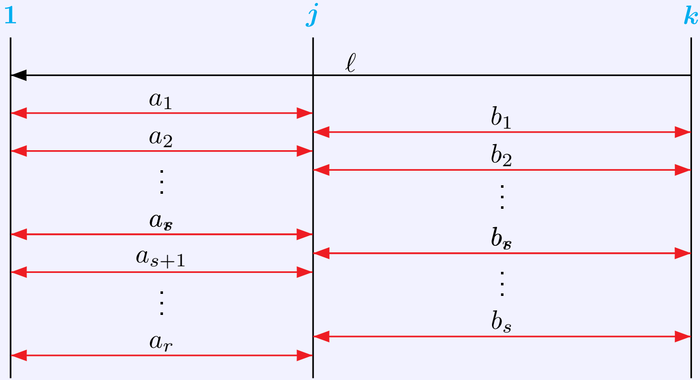

$M^j_{v_\ell, w_\ell}$ with $v_\ell = a_1 a_2 \ldots a_r$ and $w_\ell = b_1 b_2 \ldots b_s$

$$L_\Delta = M_{v_1}.\{M_{v_1}, \ldots, M_{v_t}\}^* \cup M_{w_1}.\{M_{w_1}, \ldots, M_{w_t}\}^* \cup \bigcup_j M^j_{v_1,w_1}\{M^j_{v_1,w_1}, \ldots, M^j_{v_t,w_t}\}^*$$

Undecidability: Reduction

Lemma :

The MPCP Δ has a solution iff L_Δ has some $(k-1)$-implied scenario.

Proof : Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1} v_{i_2} \ldots v_{i_m} = a_1 a_2 \ldots a_\ell = w_{i_1} w_{i_2} \ldots w_{i_m}$$

We build an MSC $M \notin L_\Delta$ which is $(k-1)$-implied by L_Δ.

Undecidability: Reduction

Proof : Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

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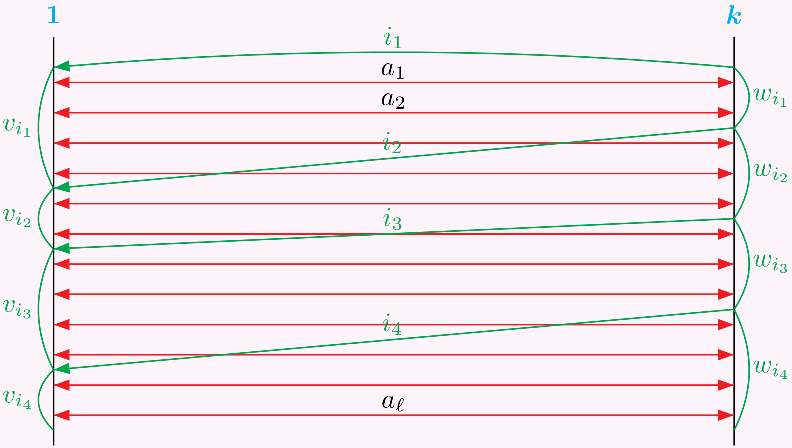

$$M \notin L_\Delta$$

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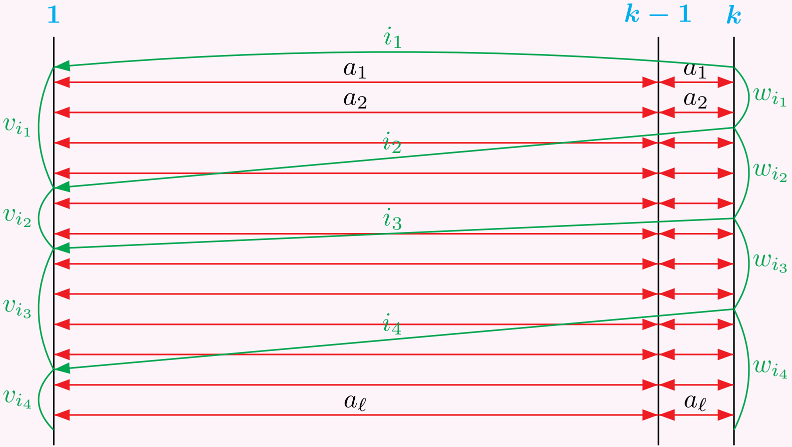

$$M\restriction_{1,\ldots,k-1} = (M_{v_{i_1}} M_{v_{i_2}} \ldots M_{v_{i_m}})\restriction_{1,\ldots,k-1}$$

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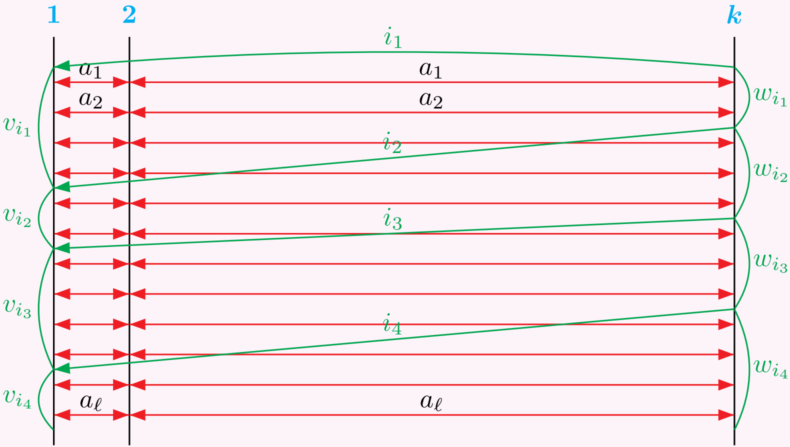

$$M\restriction_{2,\ldots,k} = (M_{w_{i_1}} M_{w_{i_2}} \ldots M_{w_{i_m}})\restriction_{2,\ldots,k}$$

Undecidability: Reduction

Proof : Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

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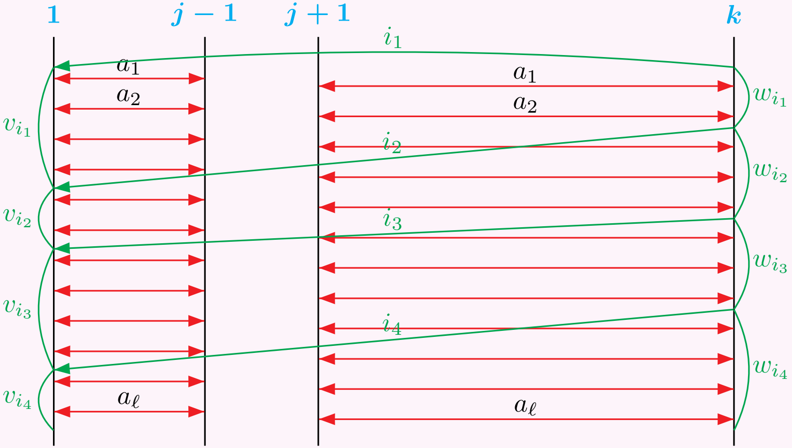

$$M\restriction_{1,\ldots,j-1,j+1,\ldots,k} = (M^j_{v_{i_1},w_{i_1}} M^j_{v_{i_2},w_{i_2}} \ldots M^j_{v_{i_m},w_{i_m}})\restriction_{1,\ldots,j-1,j+1,\ldots,k}$$

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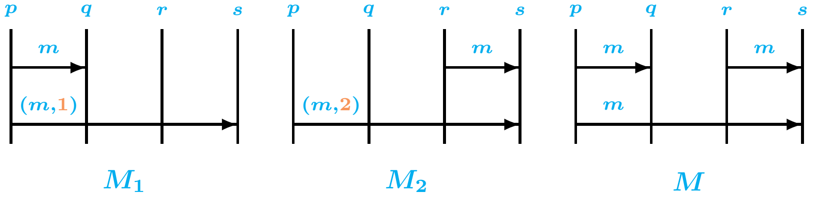
6 Decidability of 1-testability without message contents

1-testability over the singleton alphabet

- Each channel behaves as counter
- Encode the behaviours of the HMSC as a Petri Net
- HMSC defines a regular language $\Rightarrow$ channels are bounded by some constant B
  1. Check if net has $B+1$ messages in a channel en route to final marking
  2. If yes, implied scenario exists
  3. Otherwise, language of net is regular
     - Check if net exhibits any behaviour not described by HMSC
- Special case of a result due to Morin [M02]

Future work

- Local testability is undecidable in most situations
- Look for sufficient conditions that indicate violation of testability
- Testability by piggybacking auxiliary information?



By tagging auxiliary information to m,
p informs s whether it has sent a message to q
This rules out the implied scenario M

- Can we piggyback a bounded amount of auxiliary information to ensure testability?
- Bounded auxiliary information suffices to check causal closure [AMNN05]