Propositional Dynamic Logic and Asynchronous Cascade Decompositions for Regular Trace Languages

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Joint Work with Bharat Adsul (IIT Bombay), Saptarshi Sarkar (IIT Bombay) and Pascal Weil (LaBRI, Univ. Bordeaux)

Outline

- Model of Concurrency: Mazurkiewicz Traces
- A propositional dynamic logic for traces
- Asynchronous automata (Zielonka) and asynchronous labeling functions
- Cascade product / decomposition (Krohn-Rhodes)
- Conclusion

Mazurkiewicz Traces

Architecture

$\mathscr{P} = \{1,2,3\}$

$\Sigma = \{a, b, c, d, e\}$

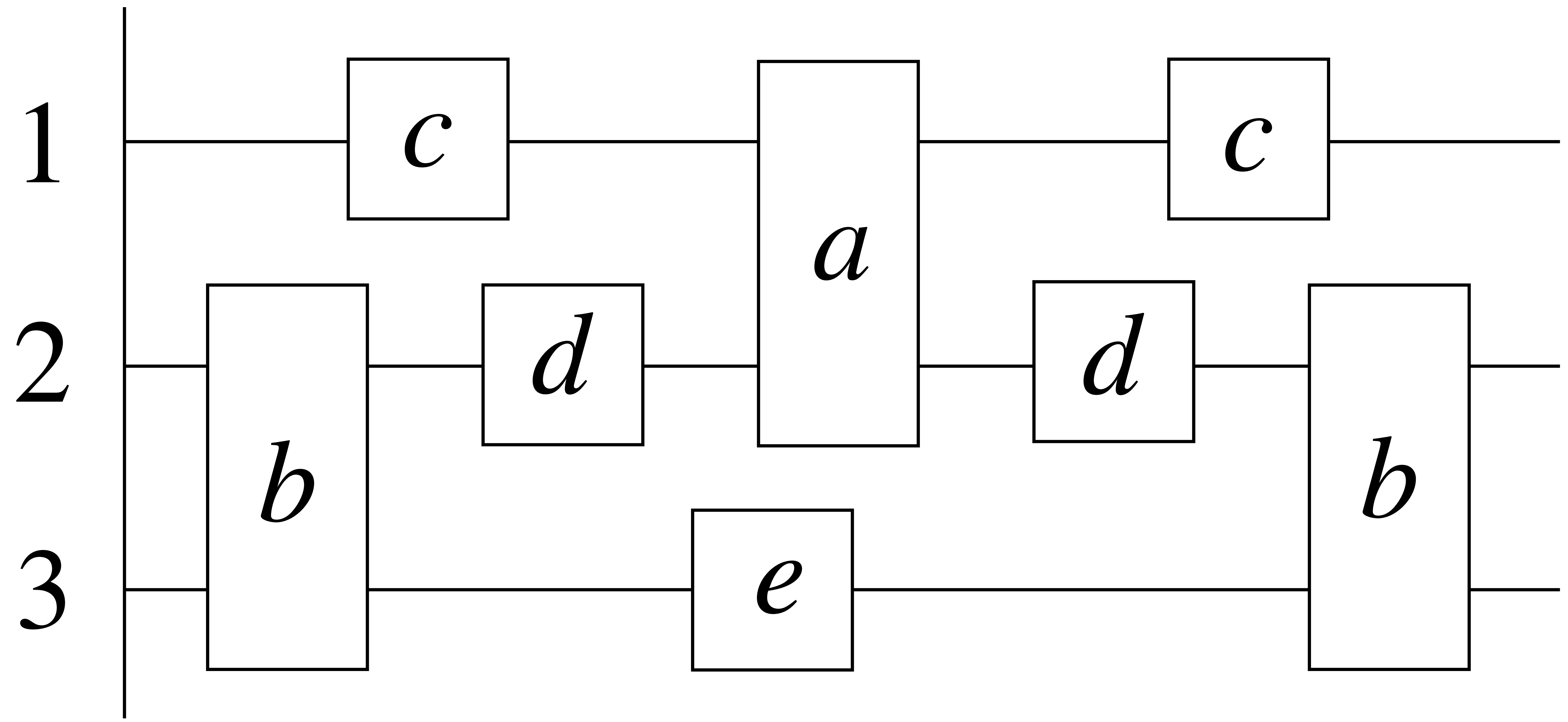
$\mathrm{loc}(a) = \{1,2\}$

$\mathrm{loc}(b) = \{2,3\}$

$\mathrm{loc}(c) = \{1\}$

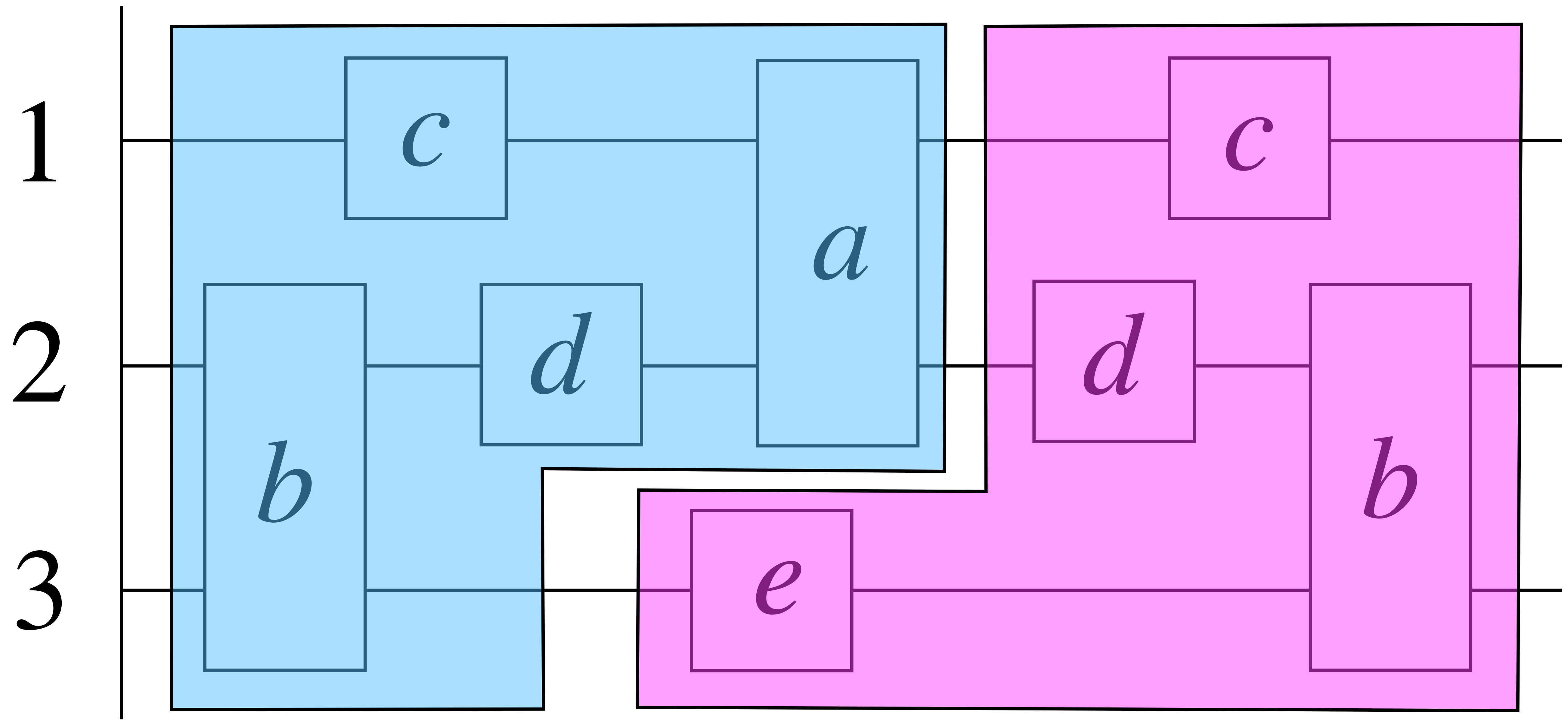
$\mathrm{loc}(d) = \{2\}$

$\mathrm{loc}(e) = \{3\}$



- set of traces denoted $Tr(\Sigma, \mathscr{P}, \mathrm{loc})$ or simply $Tr(\Sigma)$
- Trace Language: $L \subseteq Tr(\Sigma)$

Concatenation, Independence, Commutation, Monoid



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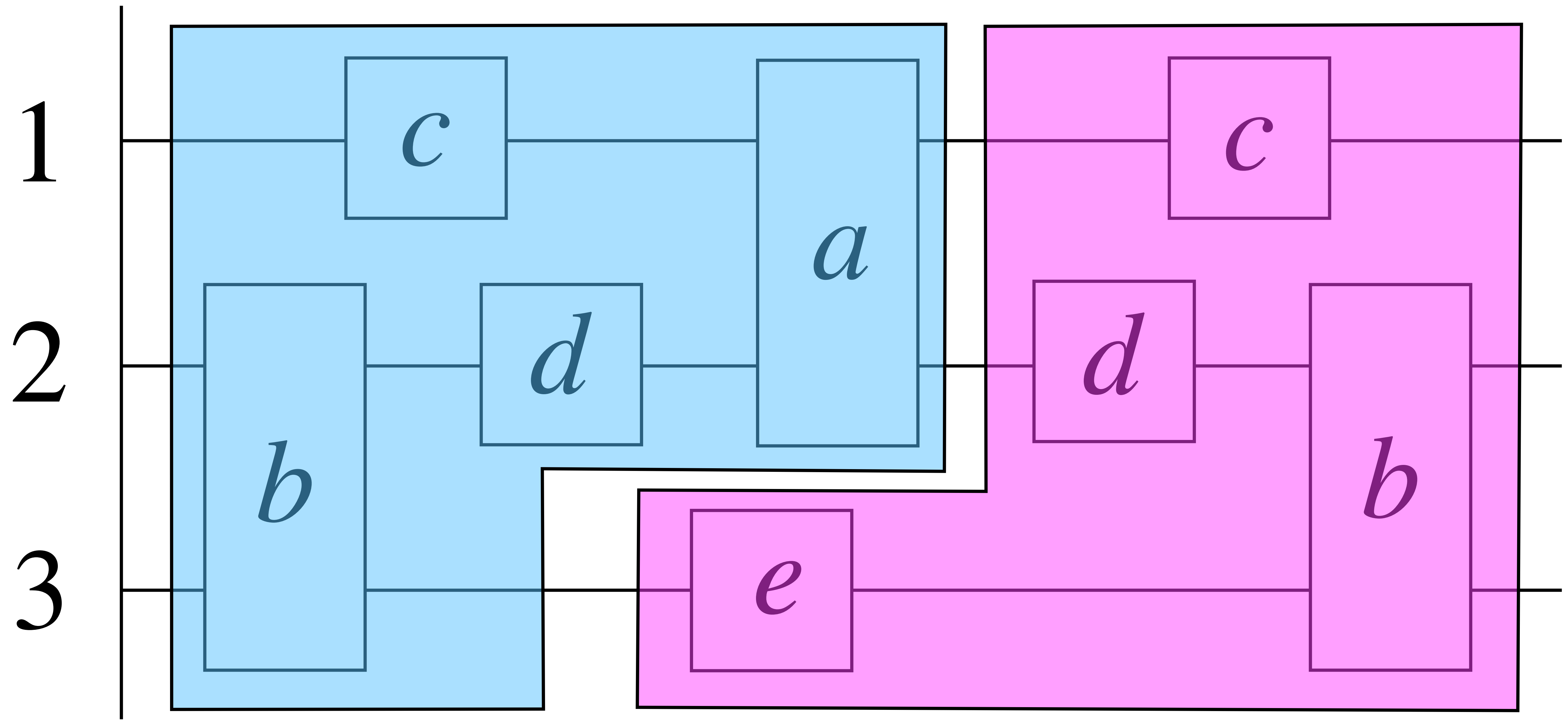

- $Tr(\Sigma)$ *with trace concatenation is a* *monoid*

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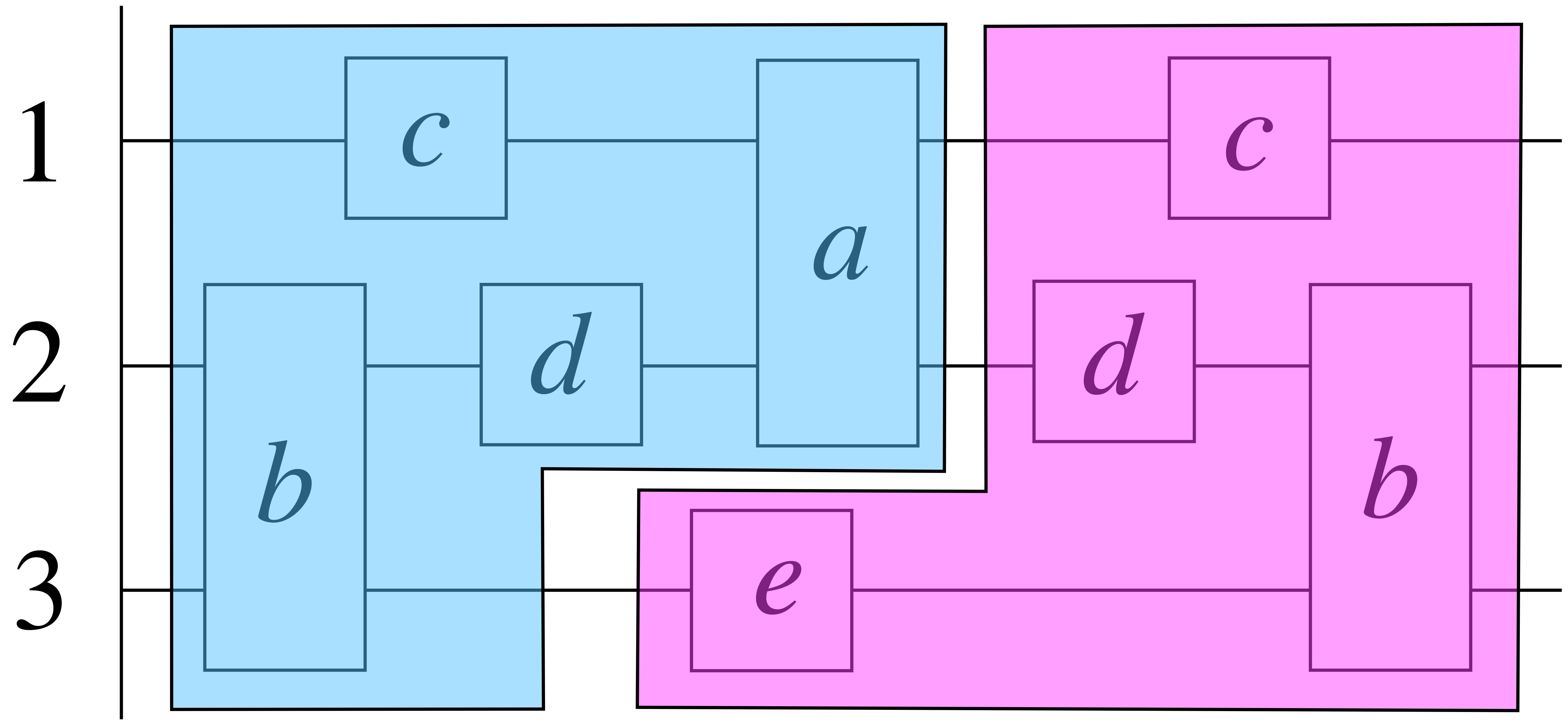
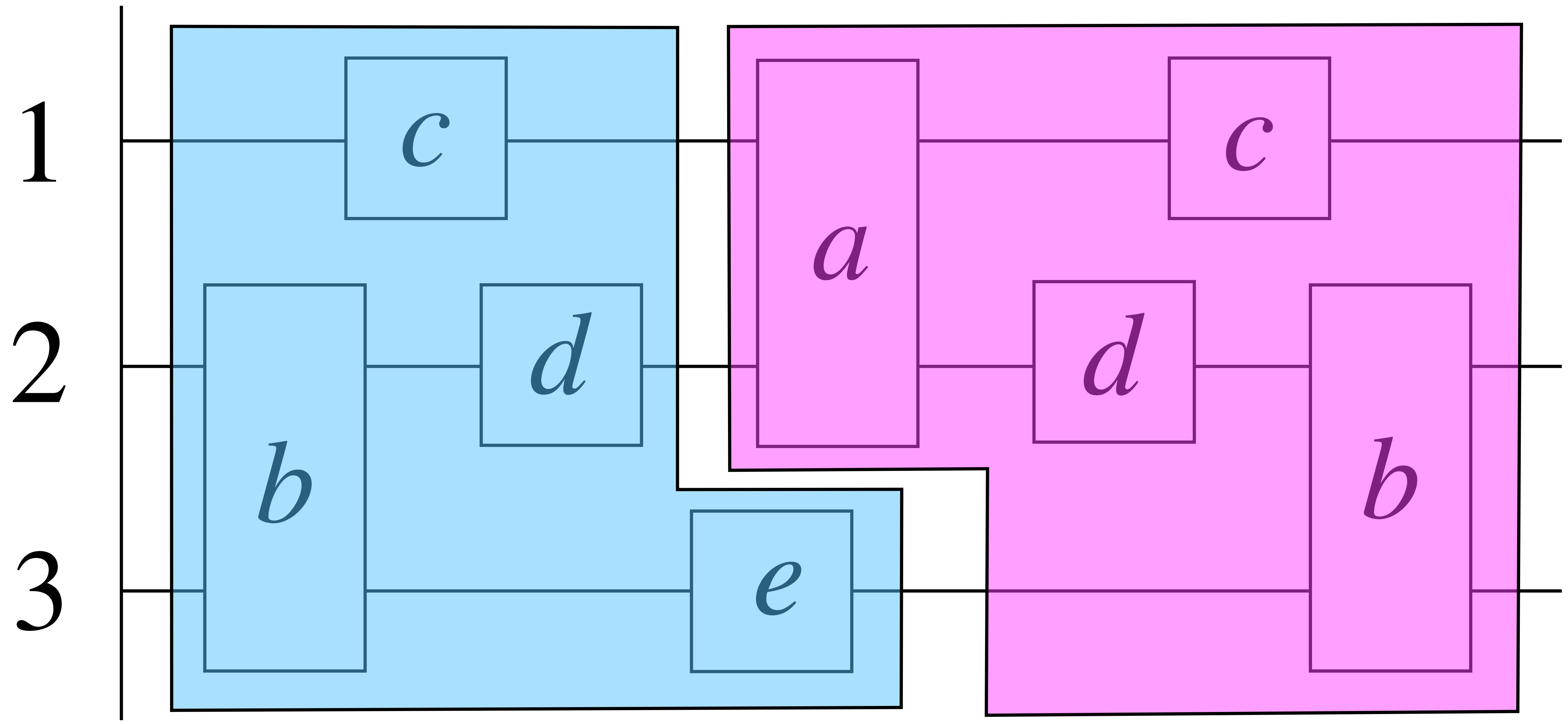


- $x \; I \; y$ iff $\mathrm{loc}(x) \cap \mathrm{loc}(y) = \emptyset$
- $a \; I \; e$
- $a \cdot e = e \cdot a$

- *$Tr(\Sigma)$ with trace concatenation is a monoid*
- *Free partially commutative monoid*

Concatenation, Independence, Commutation, Monoid



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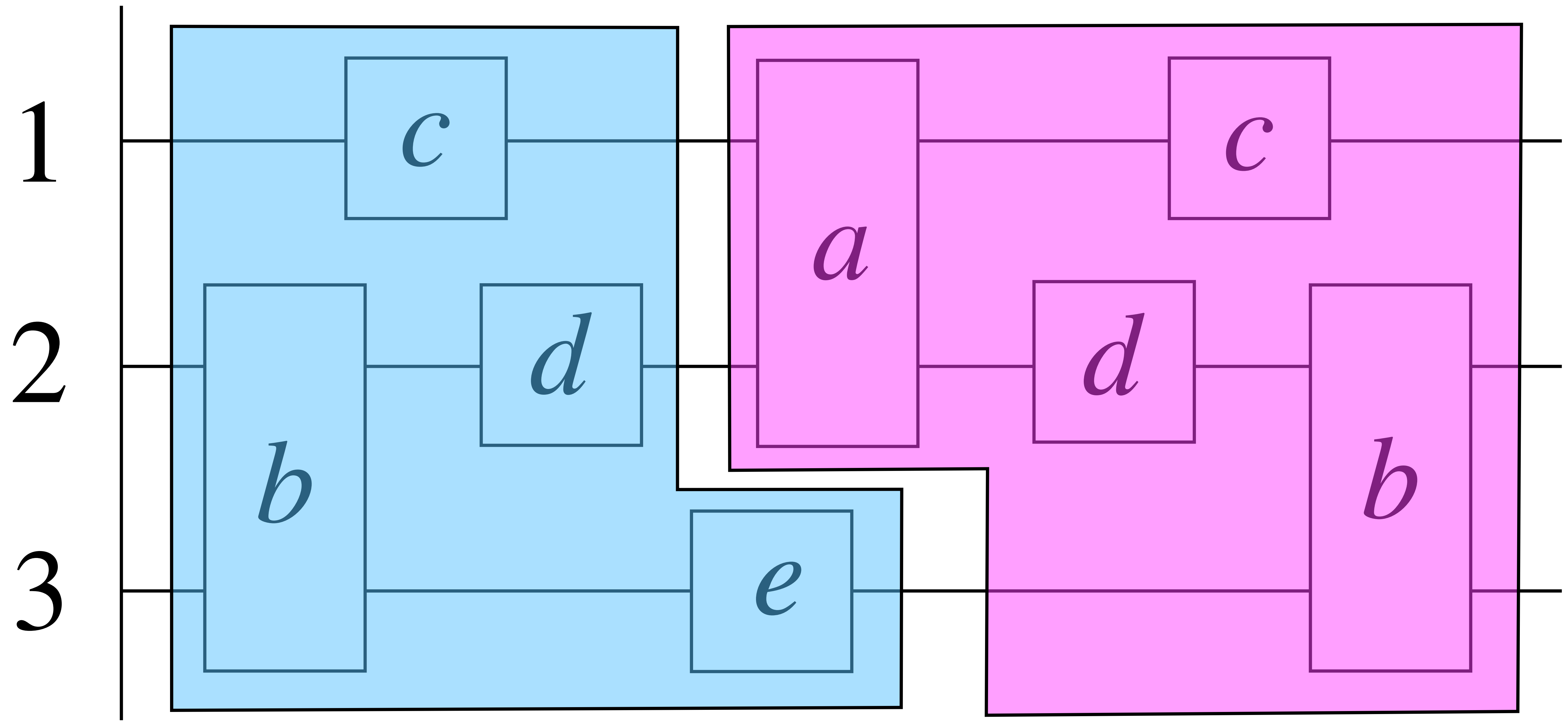

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- $Tr(\Sigma)$ *with trace concatenation is a monoid*
- *Free partially commutative monoid*
- $L \subseteq Tr(\Sigma)$ *is regular if* $L = \eta^{-1}(\eta(L))$ *for some morphism* $\eta \colon Tr(\Sigma) \to M$ *(finite monoid)*

	Expressions	Computation	Logic	Algebra
Regular Trace Languages	*Regular expressions with restricted star operation*	*Diamond automata*	*Monadic second order logic*	*Morphisms to finite Monoids*

Ochmański   Muscholl   Diekert   Gastin   Restivo   Salemi   Thomas   et. al.

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Regular Trace Languages	*Regular expressions with restricted star operation*	*Diamond automata*	*Monadic second order logic*	*Morphisms to finite Monoids*
$\supsetneq$				
Aperiodic Trace Languages	*Star-free expressions*	*Counter-free diamond automata*	*First order logic* *Temporal logic*	*Aperiodic Monoids*

Ochmański Muscholl Diekert Gastin Restivo Salemi Thomas et. al.

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Propositional Dynamic Logic

Propositional Dynamic Logic

- First introduced to reason about programs (Fischer, Ladner 1979)
    - State formulas $\varphi ::= p \mid \varphi \vee \varphi \mid \neg\varphi \mid \langle\pi\rangle\varphi$
    - Program expressions $\pi ::= \varphi? \mid x := e \mid \pi + \pi \mid \pi \cdot \pi \mid \pi^*$

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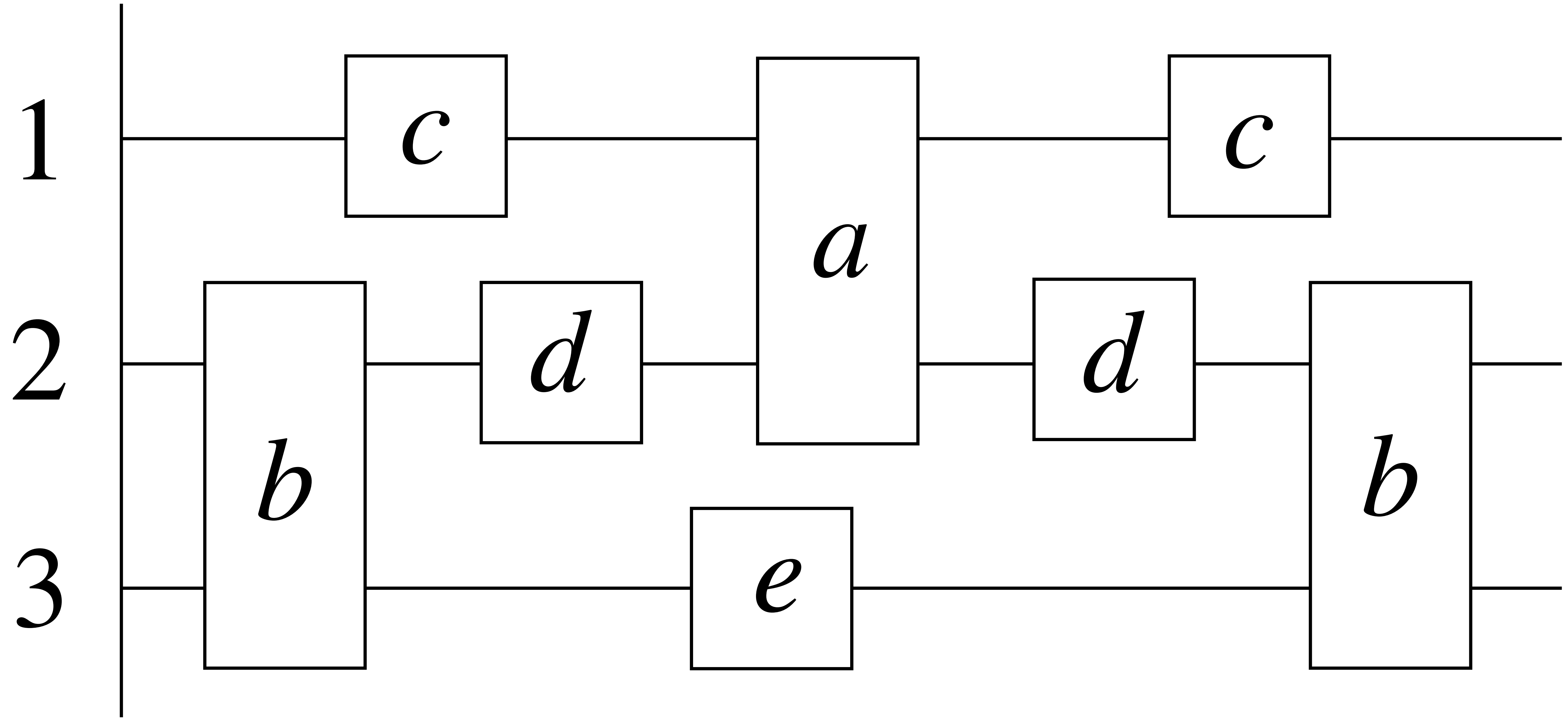
- Linear Dynamic Logic (Giacomo, Vardi 2013) over words

Regular word languages = MSO definable = LDL definable

Past PDL for Traces



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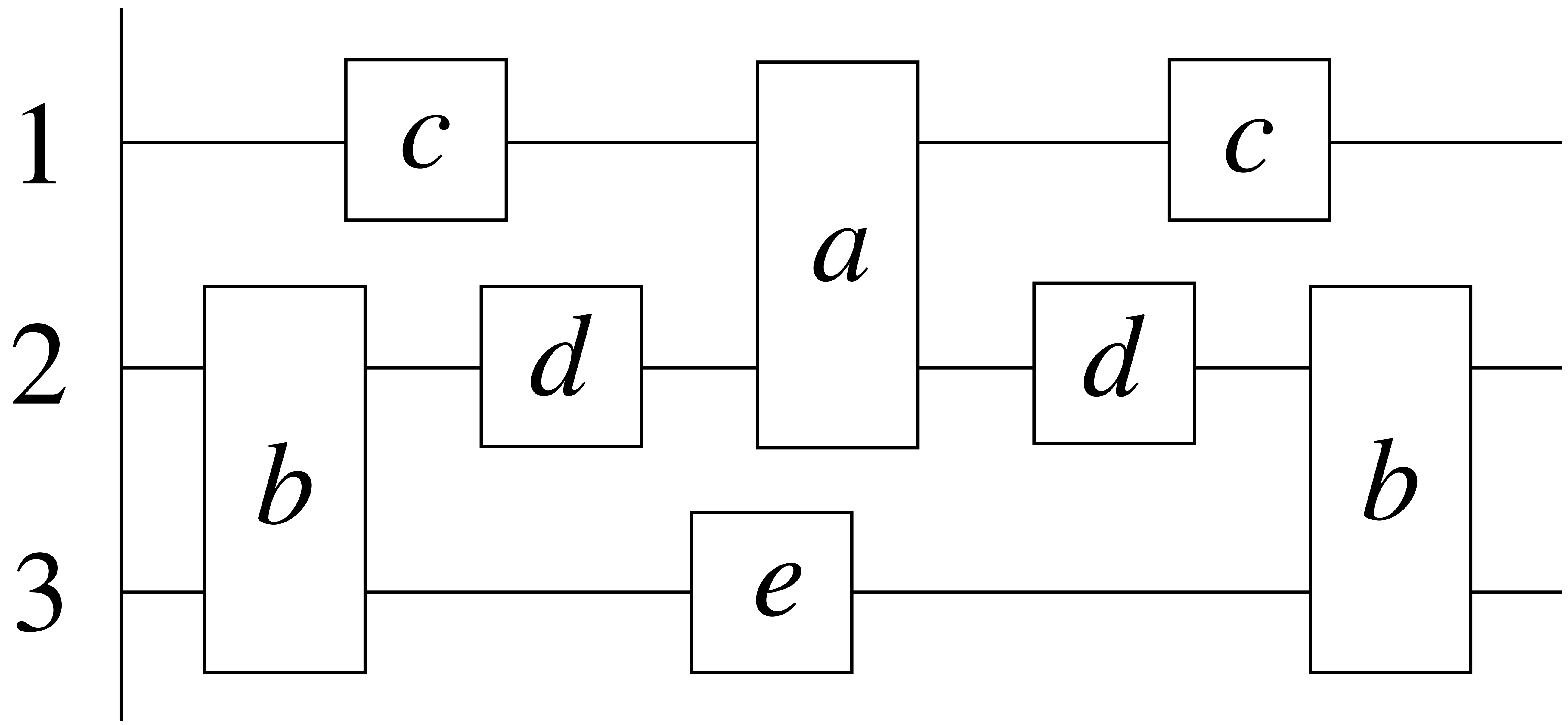

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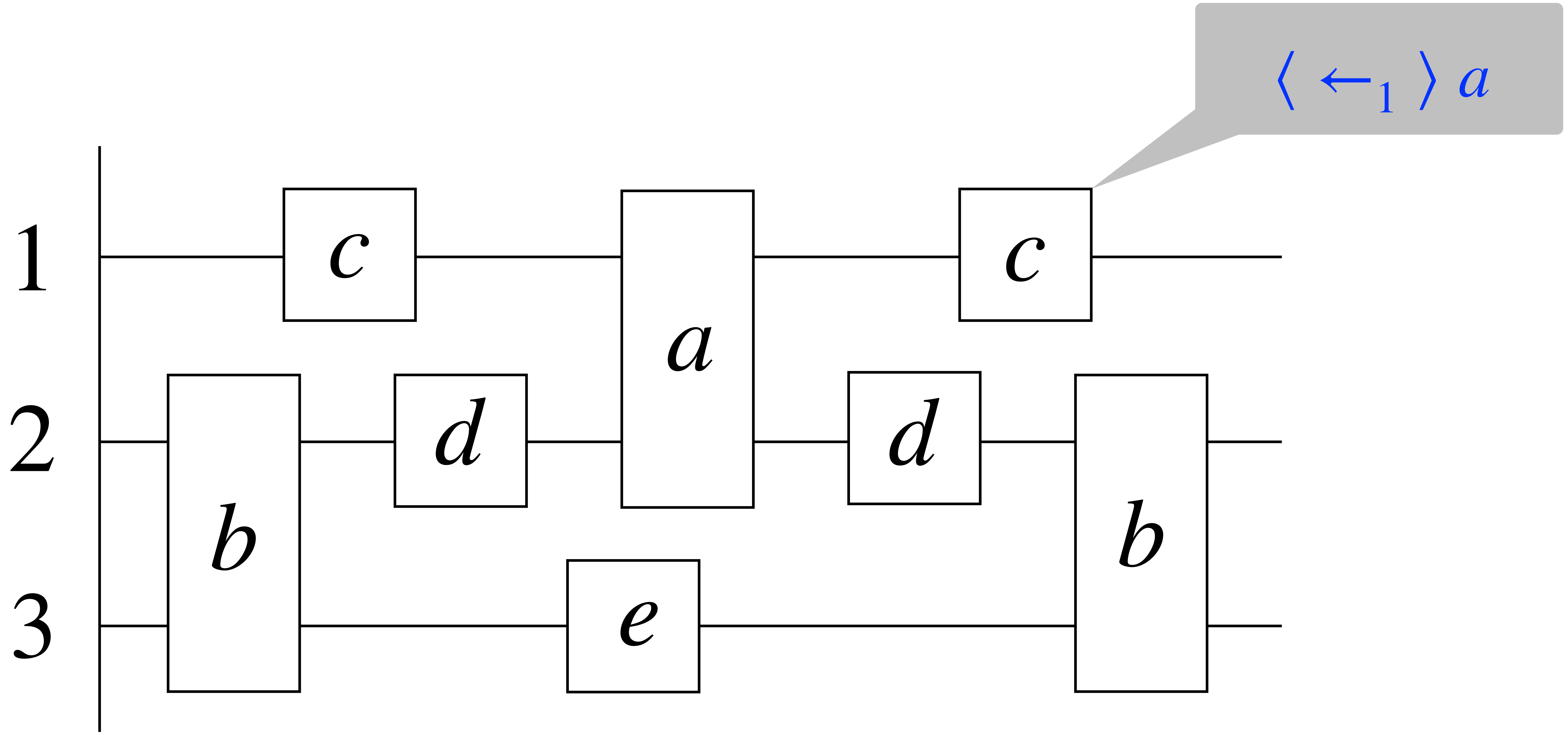

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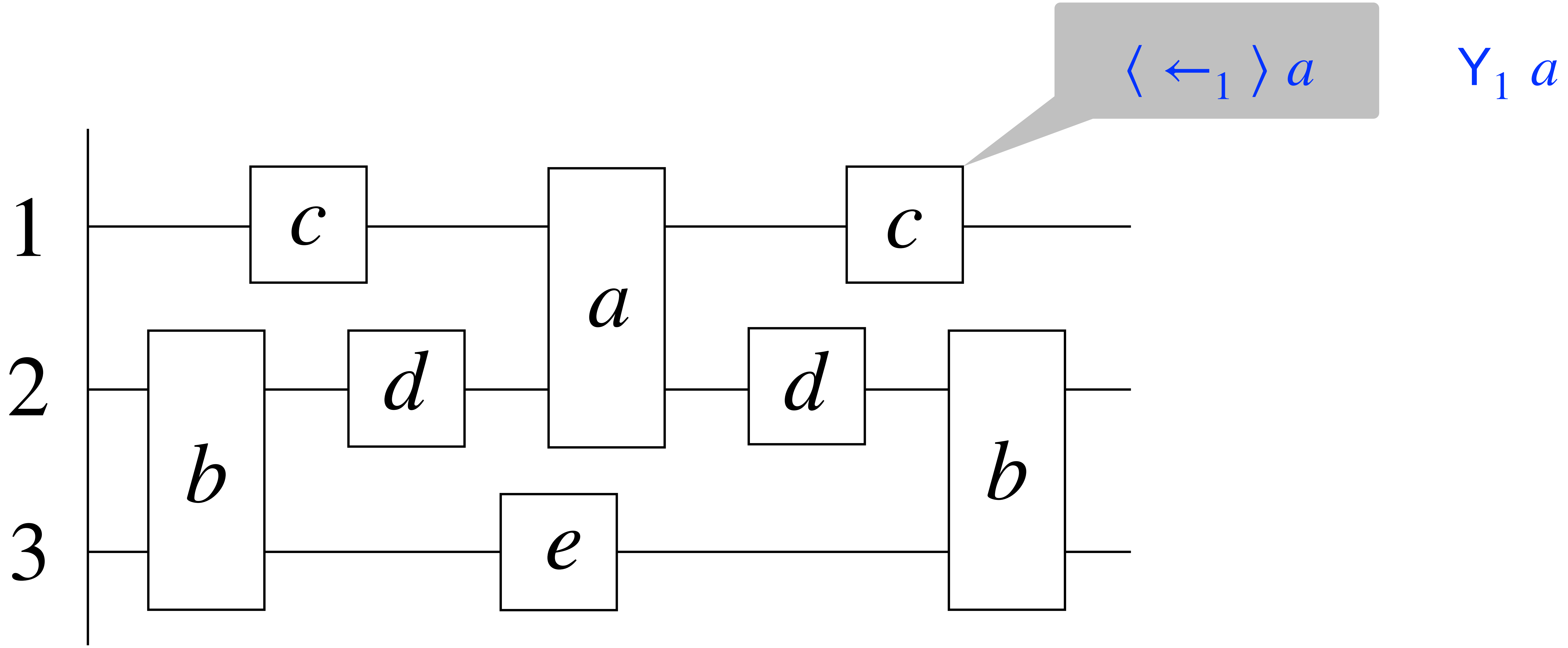

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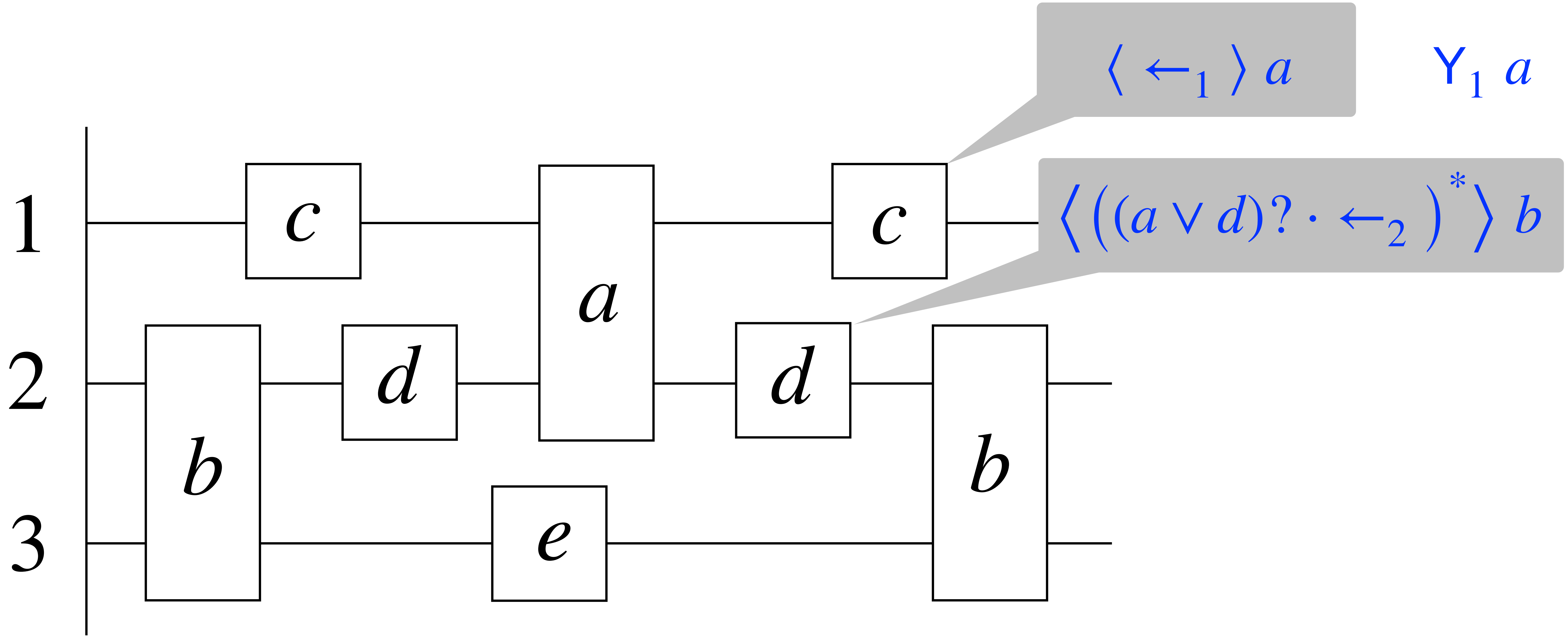

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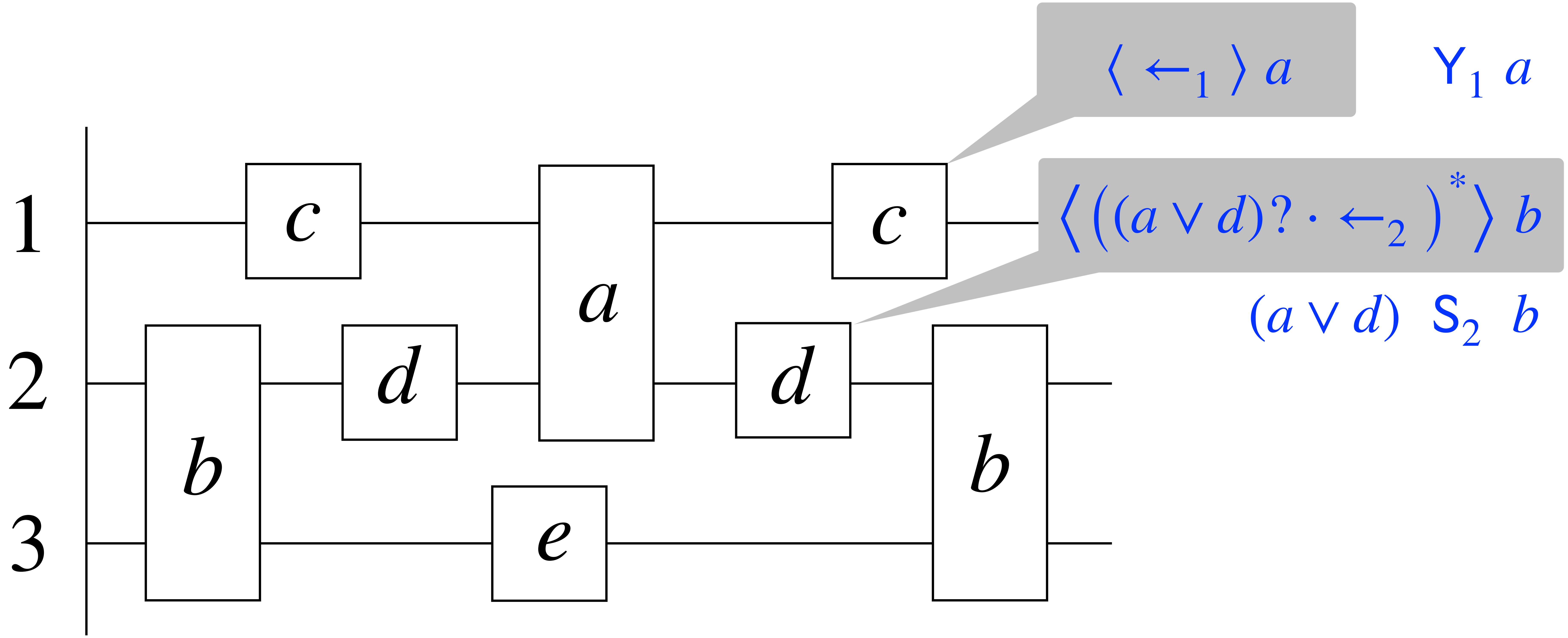

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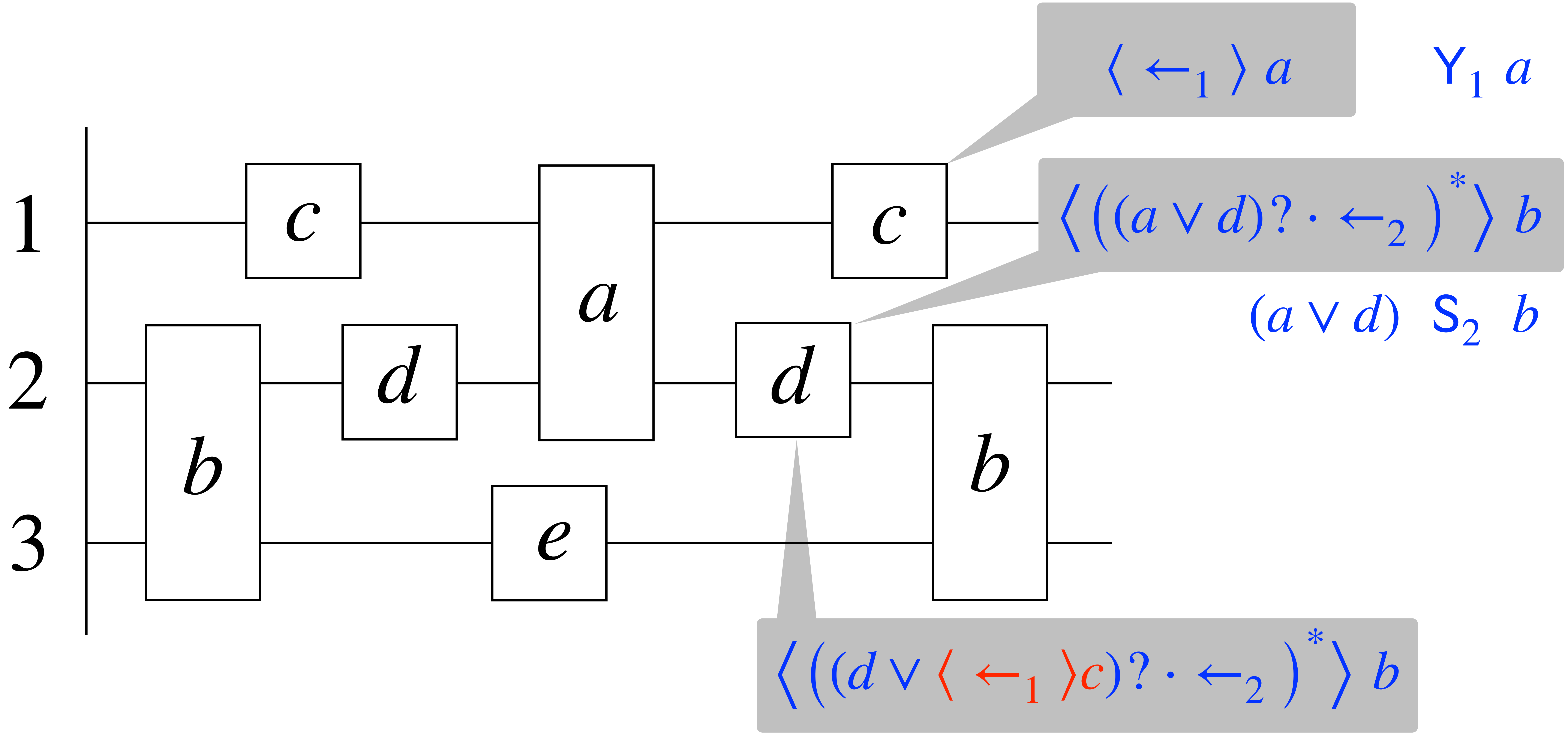

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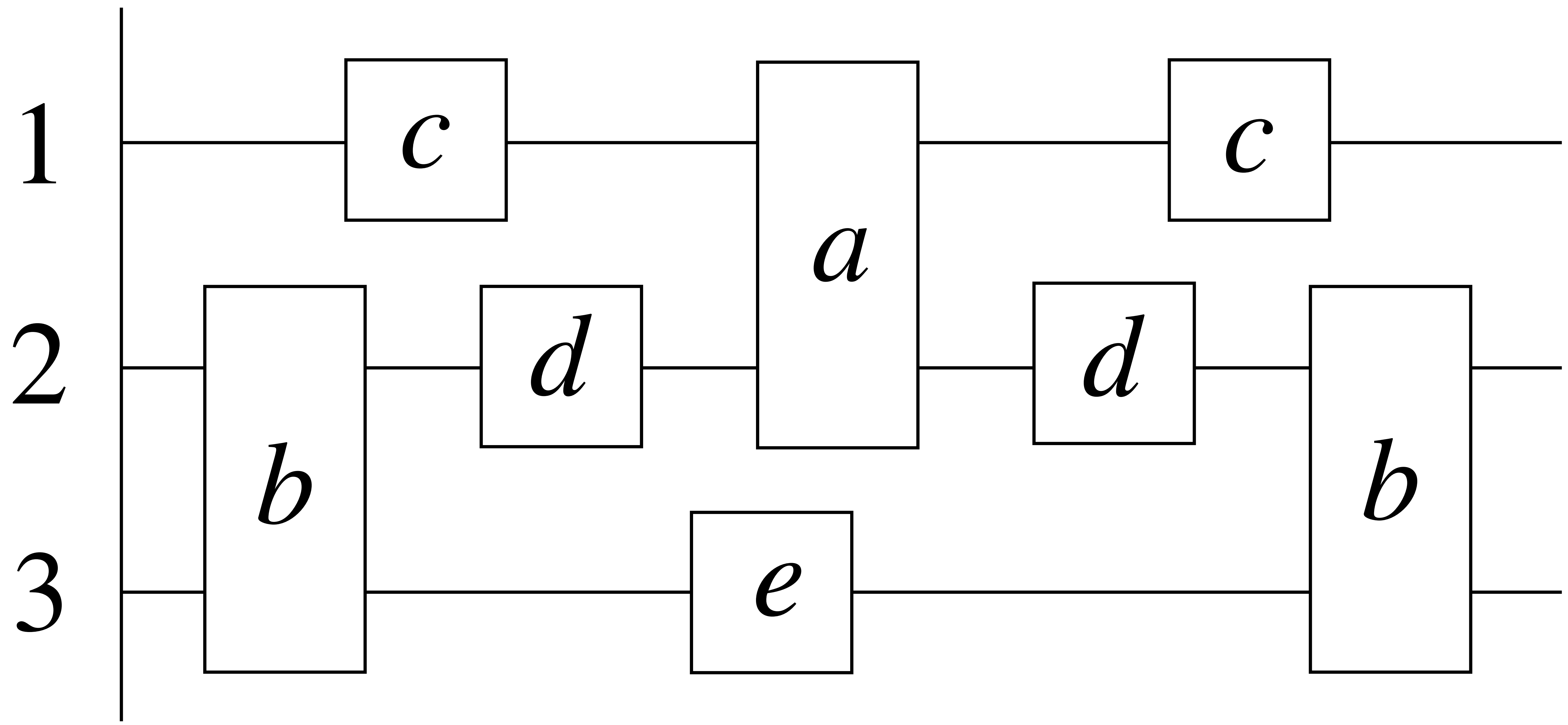

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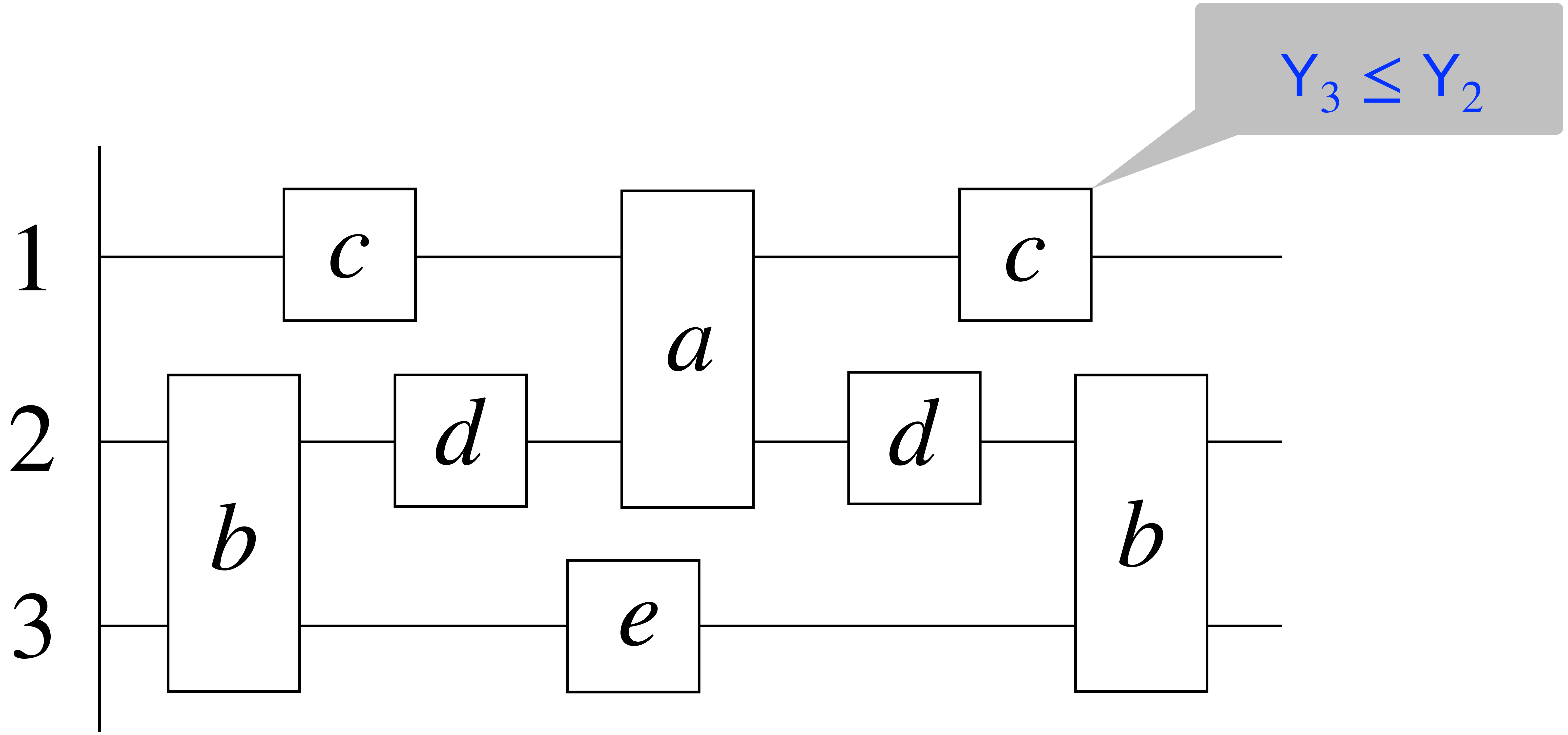

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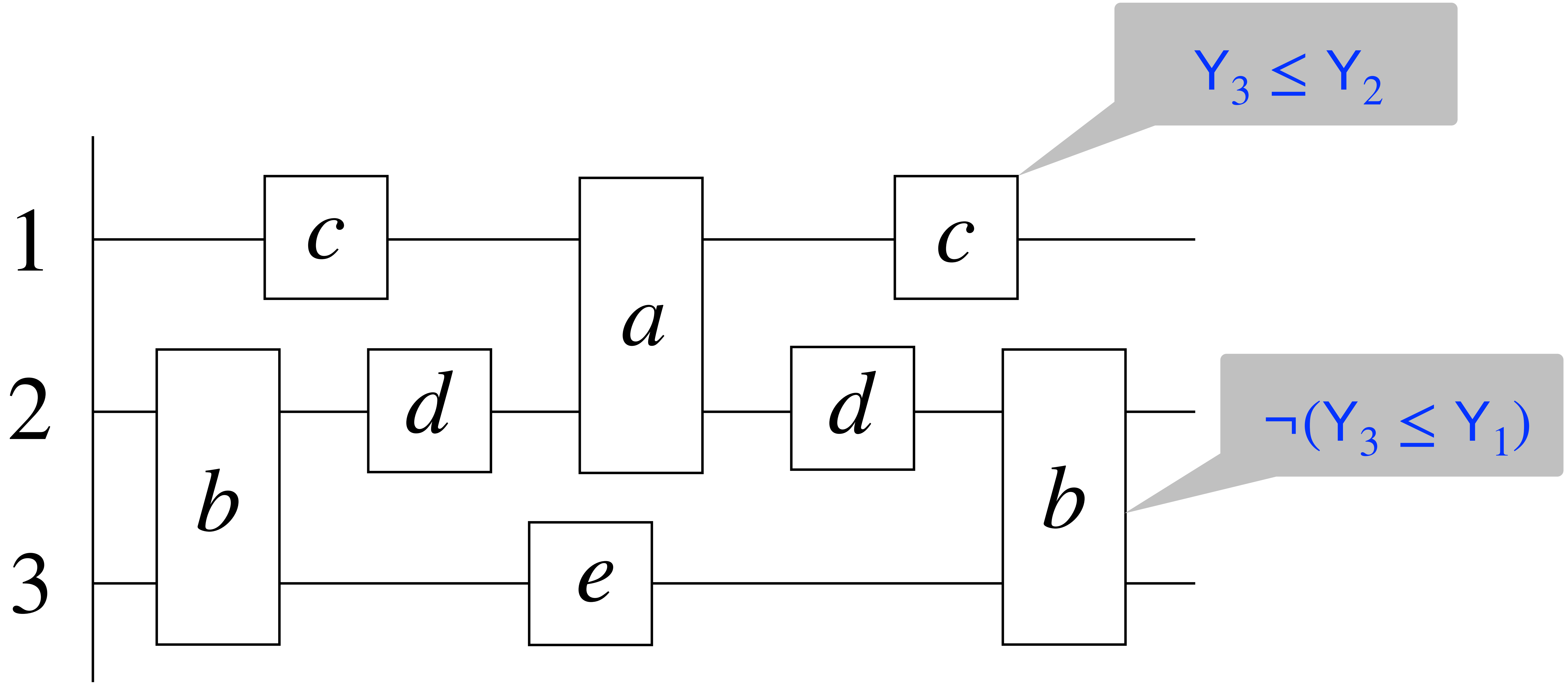

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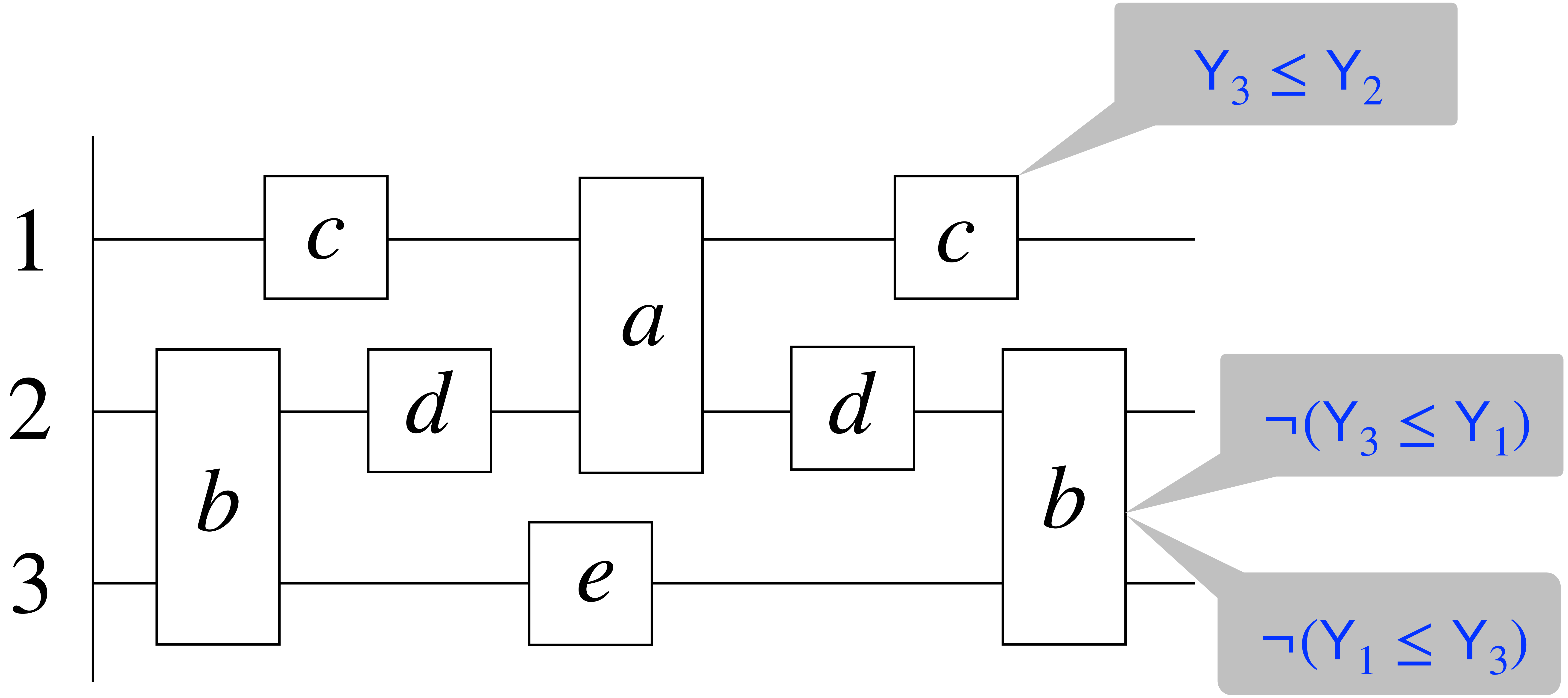

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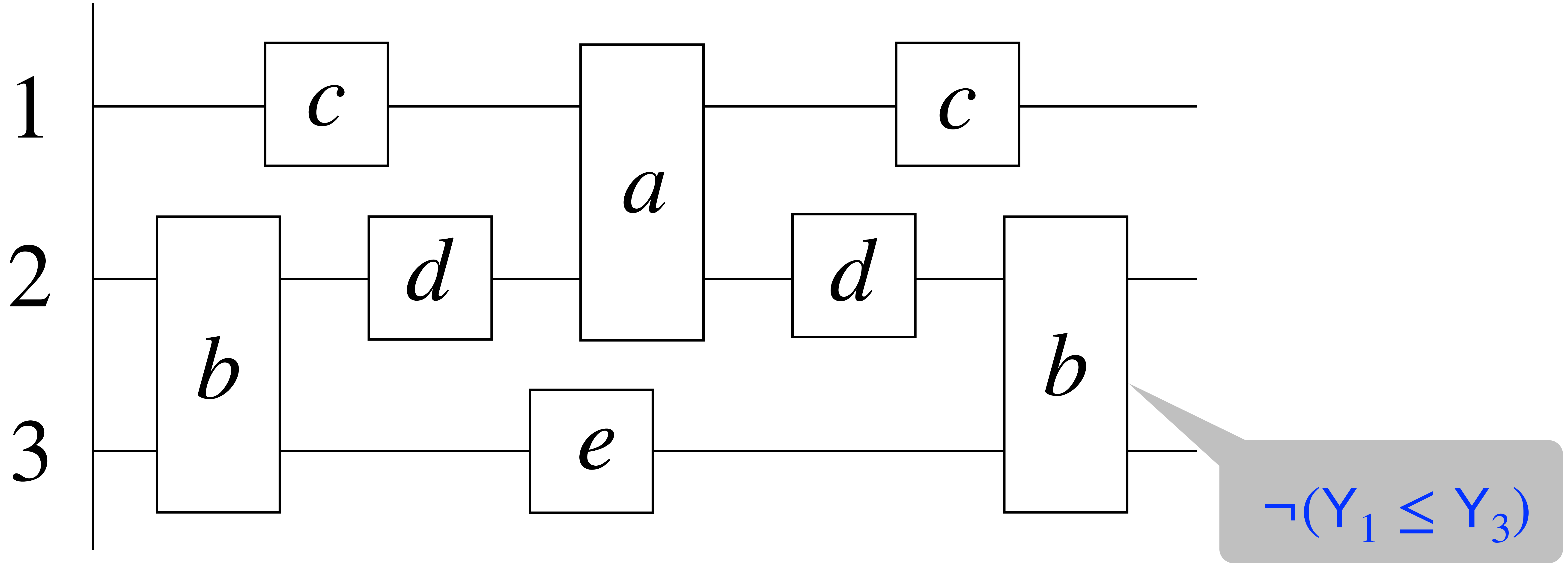

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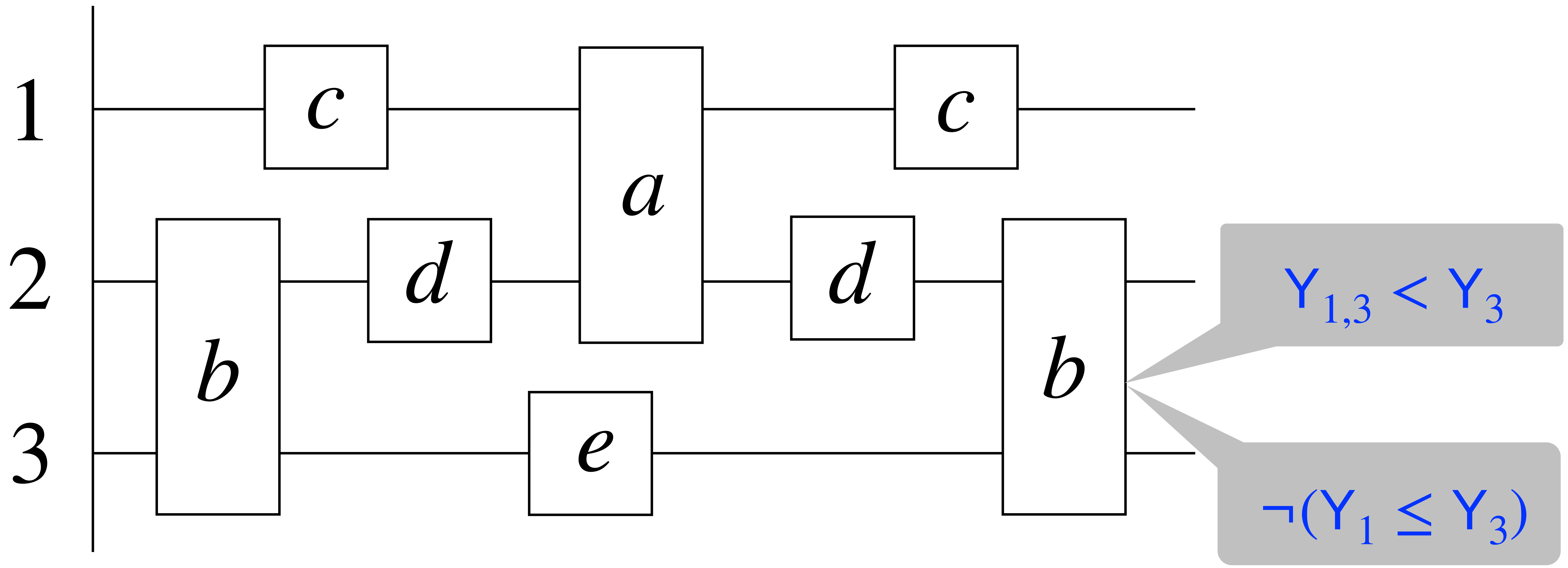

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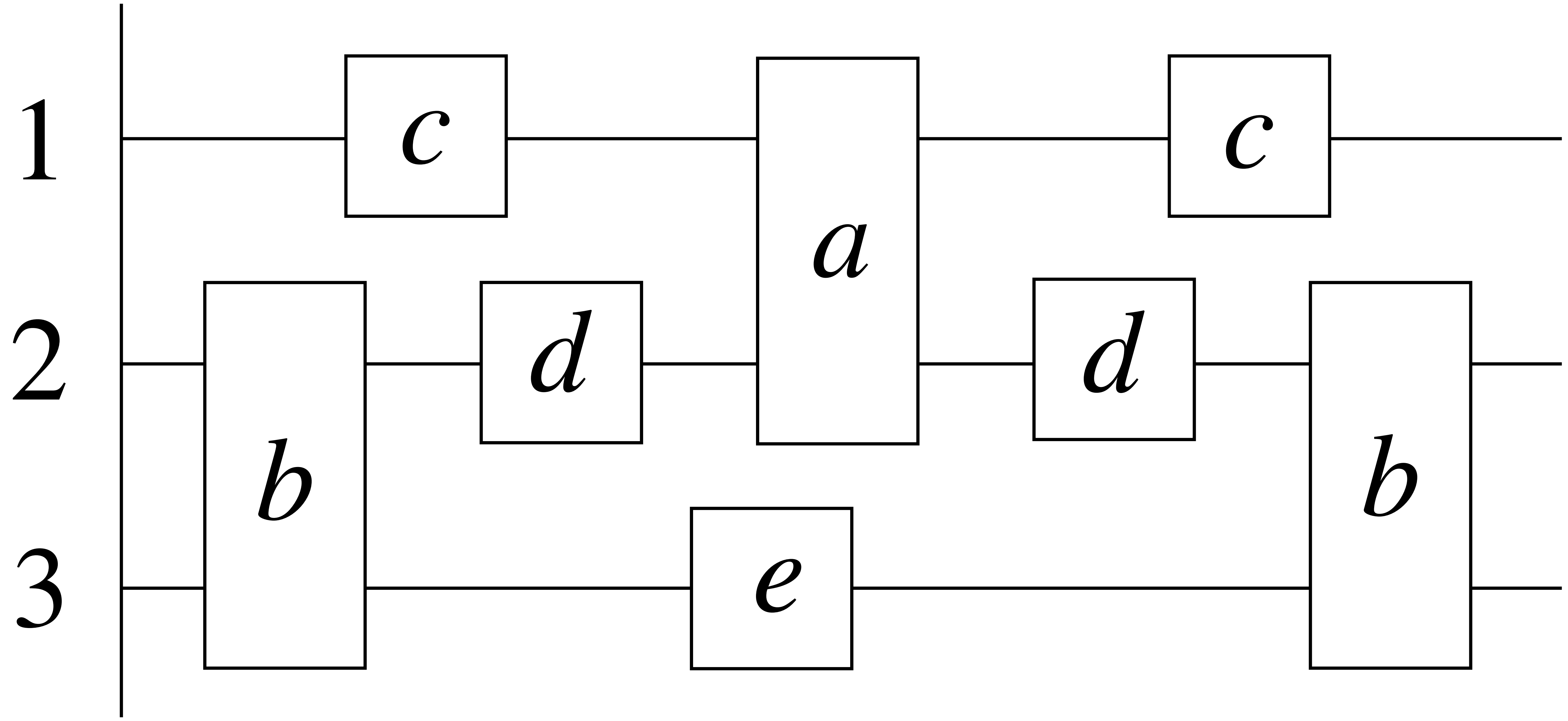

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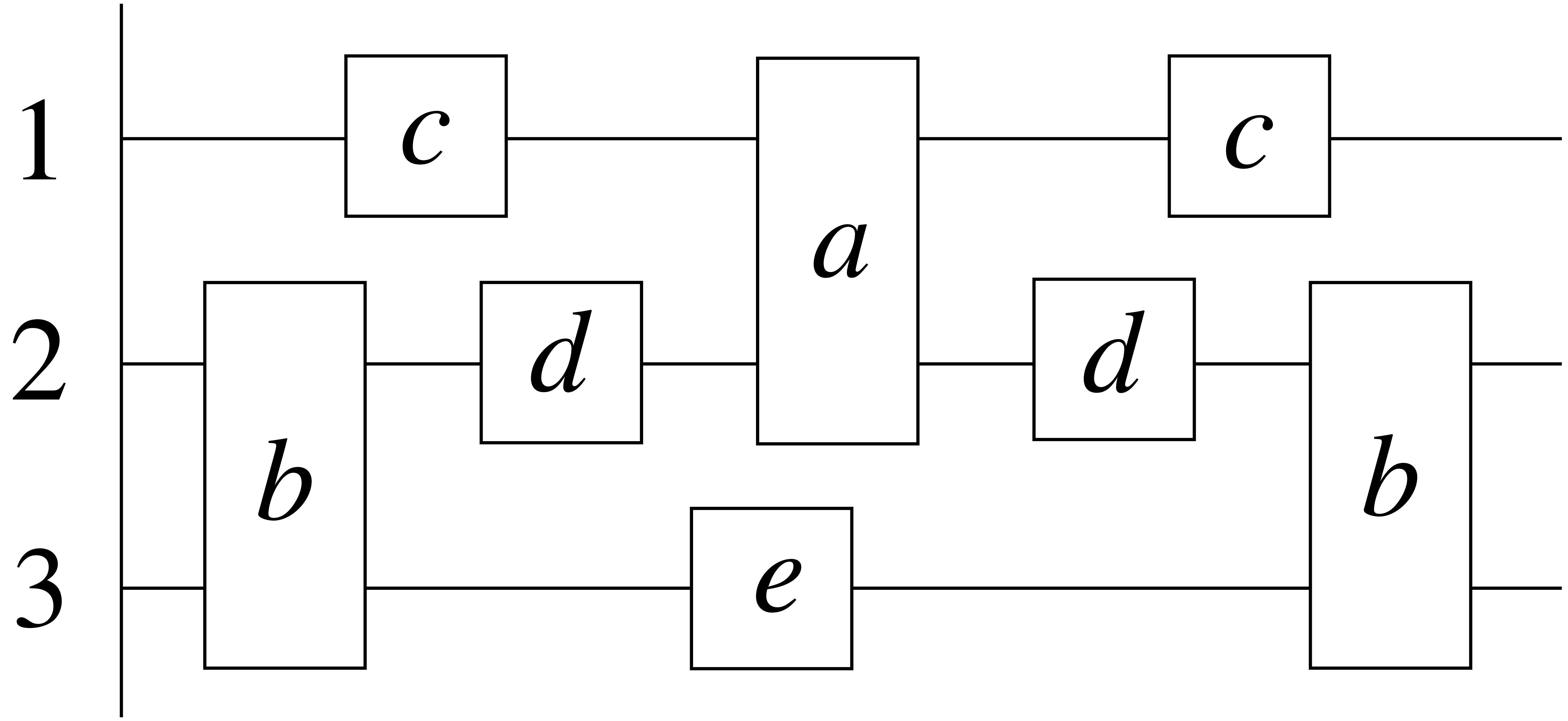
Past PDL for Traces

- $T \models \mathsf{EM}_1\, c$



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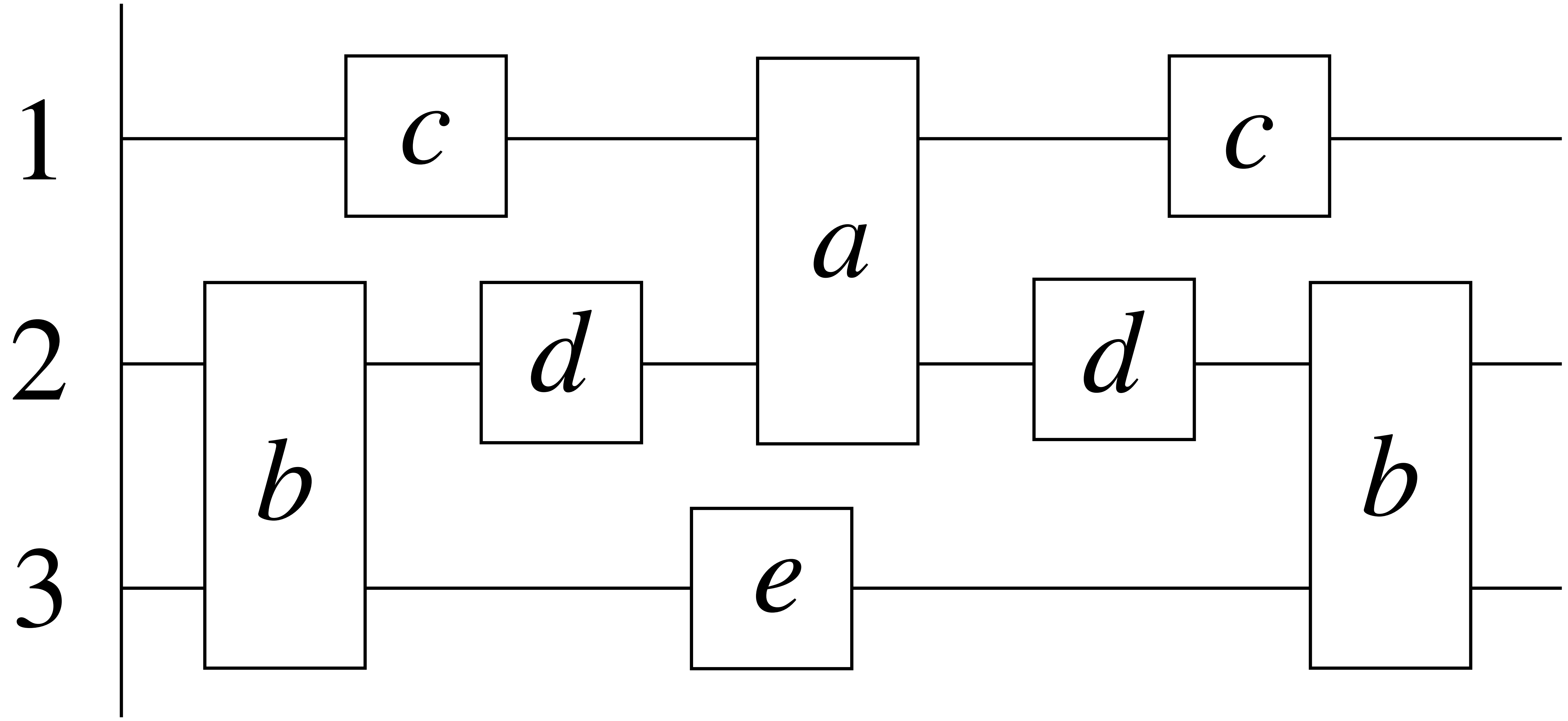

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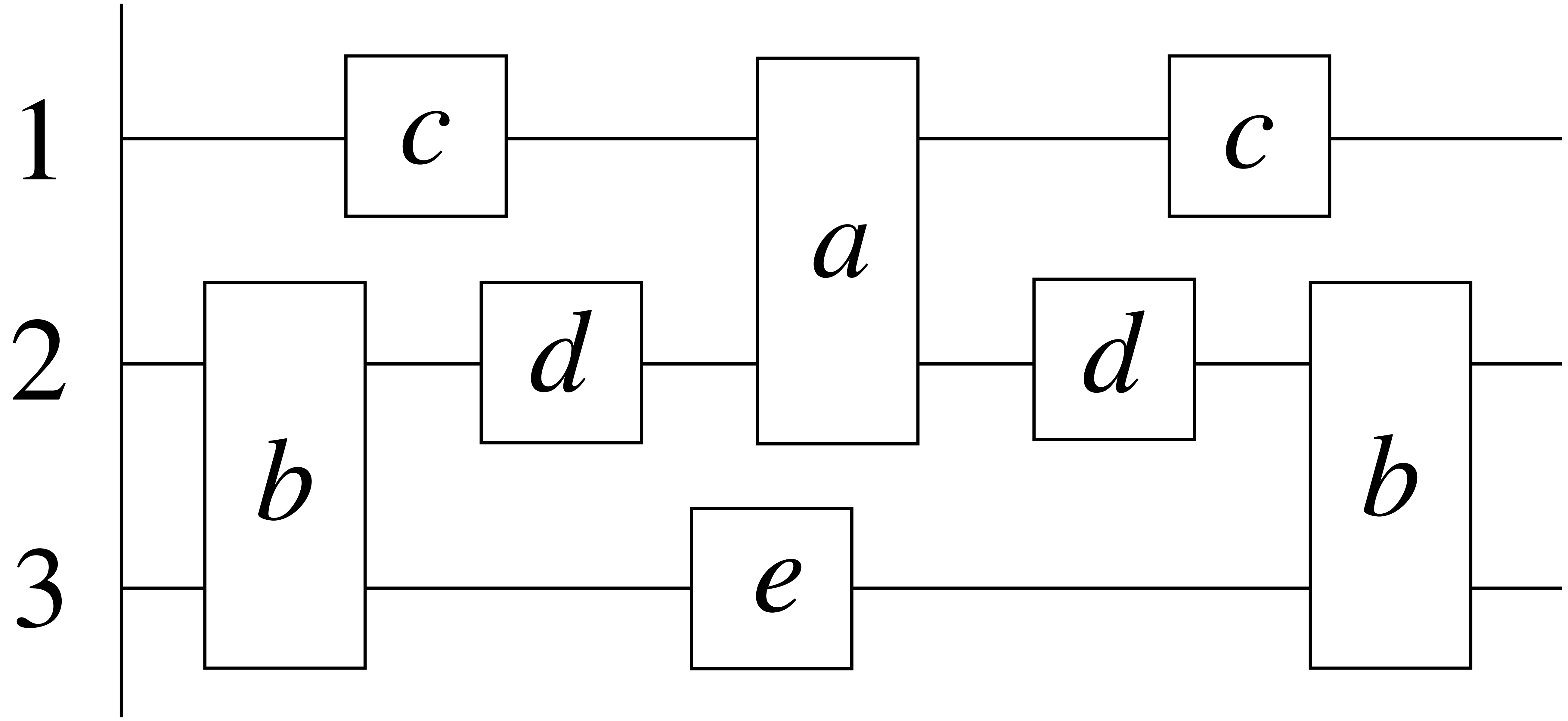

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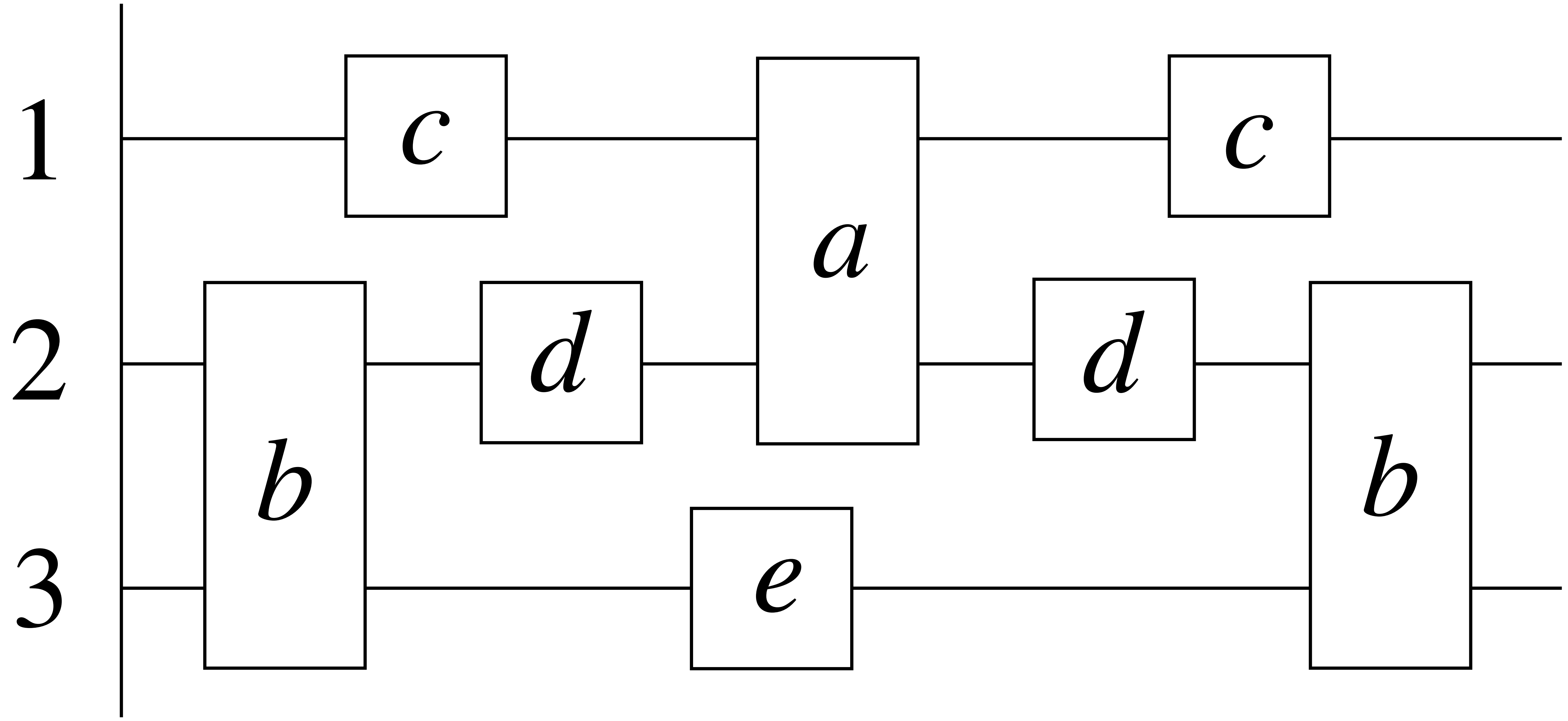
- $T \models \mathsf{L}_{1,2} < \mathsf{L}_3$



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PastPDL = Regular Trace Languages

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- Let $\eta : Tr(\Sigma) \to M$ be a morphism to a finite monoid
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  (induction on the number of processes, depends crucially on $Y_i \leq Y_j$, $Y_{i,k} \leq Y_j$)

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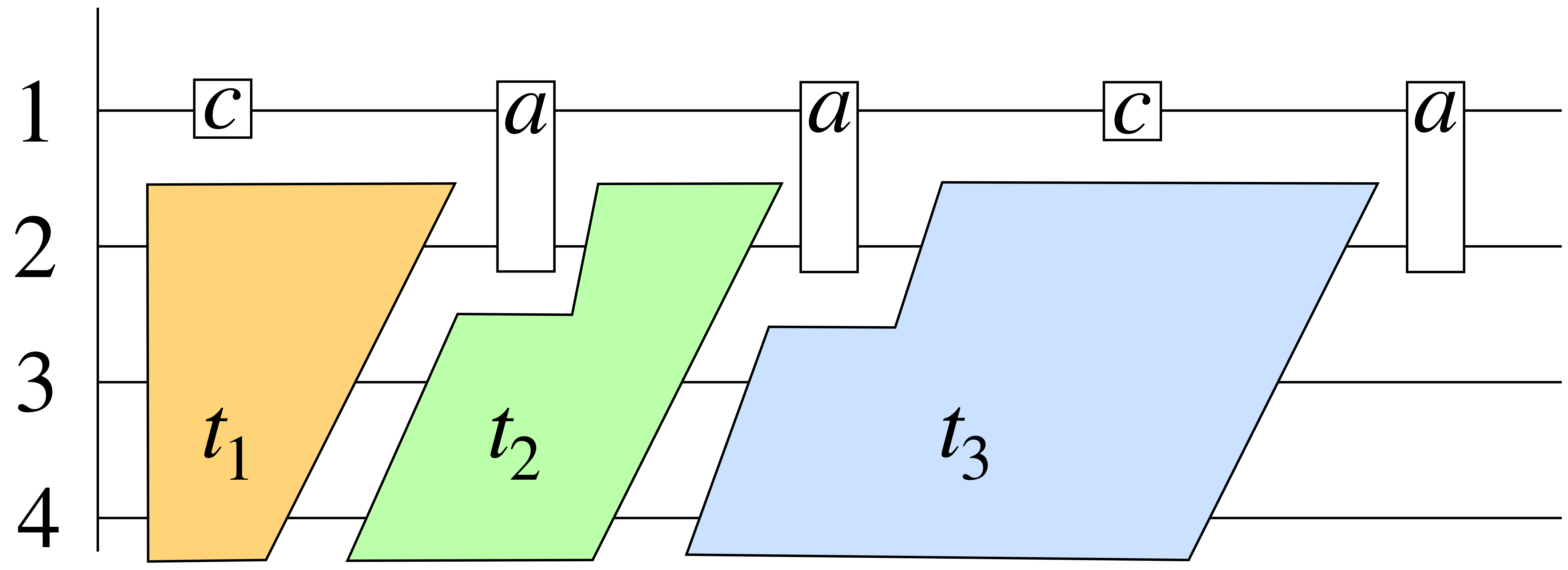

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- For the general case, decompose an arbitrary (non prime) trace into a product of prime traces.

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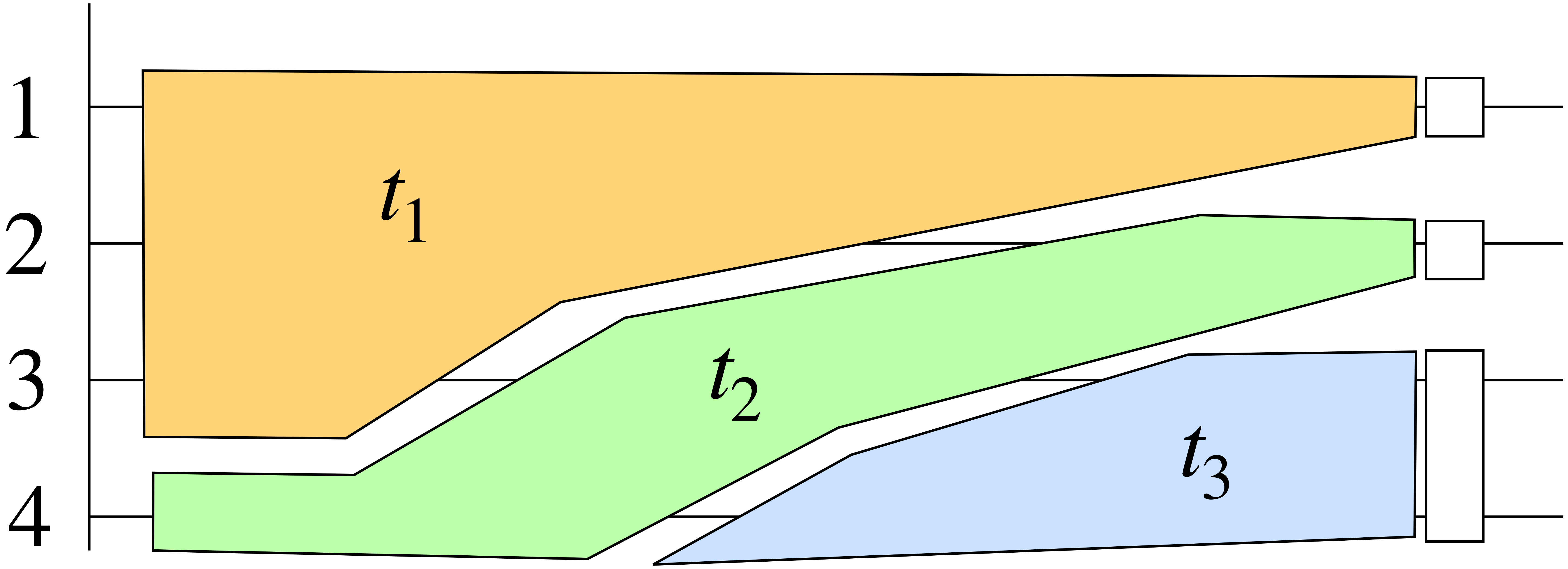

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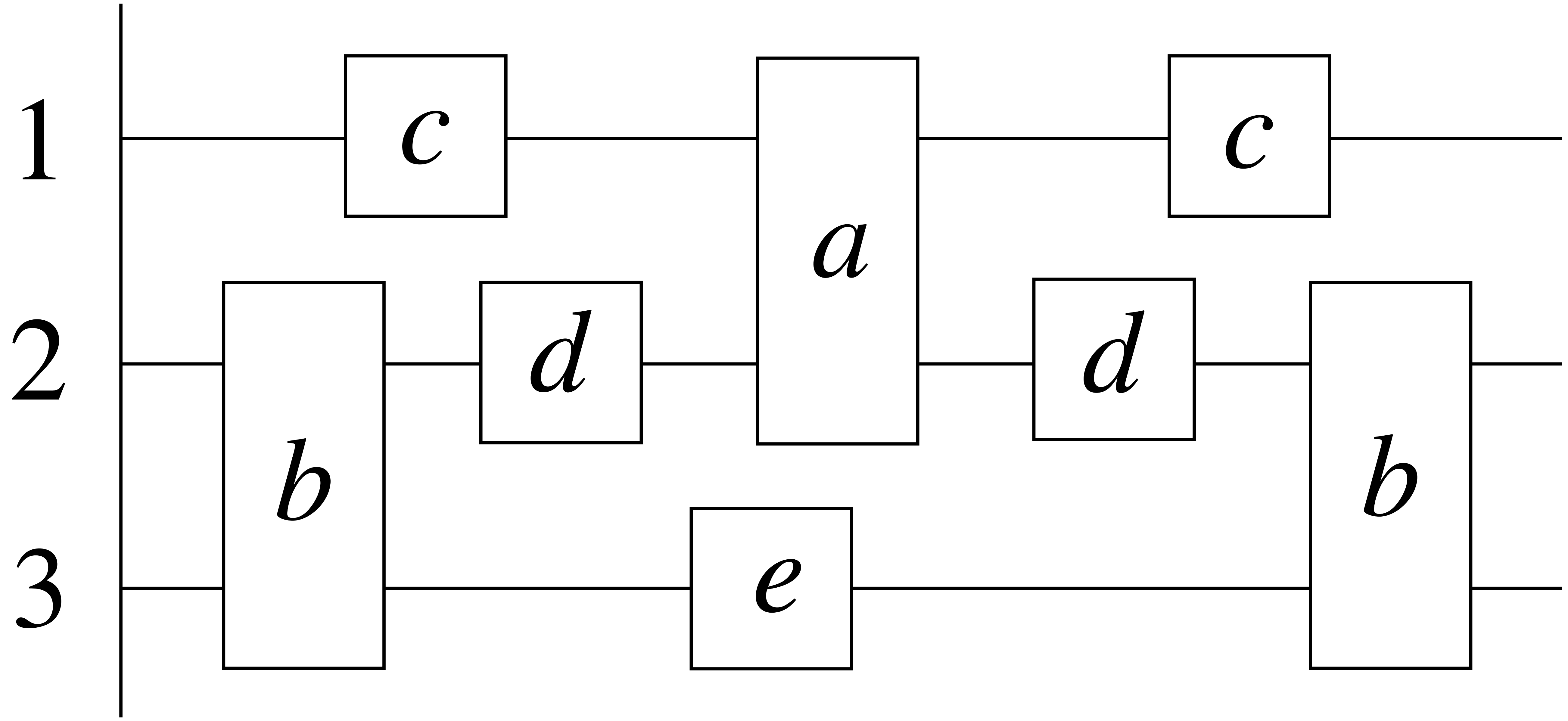
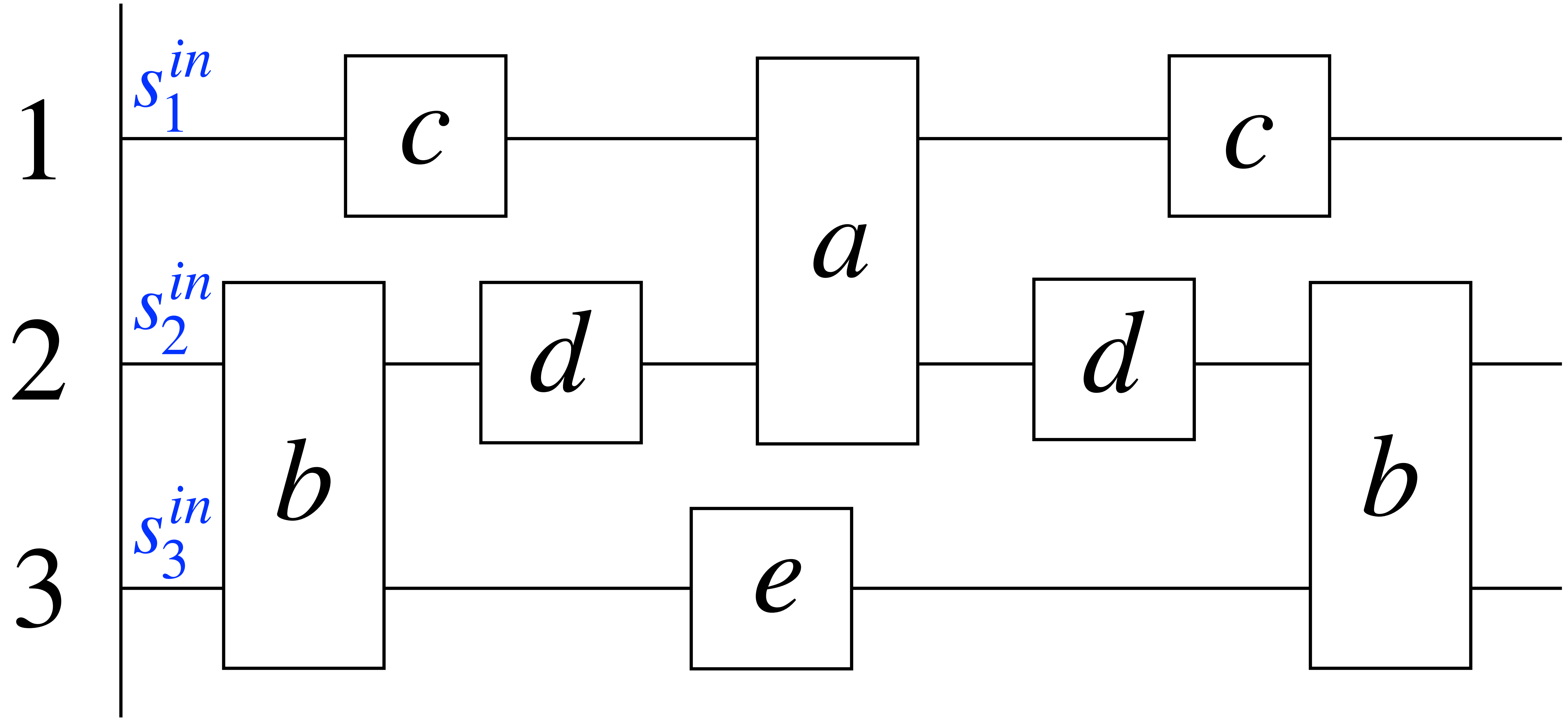
Asynchronous automata (Zielonka)



$\mathscr{A} = \left(\{S_i\}_{i\in\mathscr{P}}, \{\delta_a\}_{a\in\Sigma}, s^{in}, F\right)$

- S_i local states for process i
- δ_a transition function for action a
- $s^{in} = (s_1^{in}, s_2^{in}, s_3^{in})$ global initial state
- F global accepting states

Asynchronous automata (Zielonka)



$$\mathscr{A} = \left(\{S_i\}_{i\in\mathscr{P}}, \{\delta_a\}_{a\in\Sigma}, s^{in}, F\right)$$

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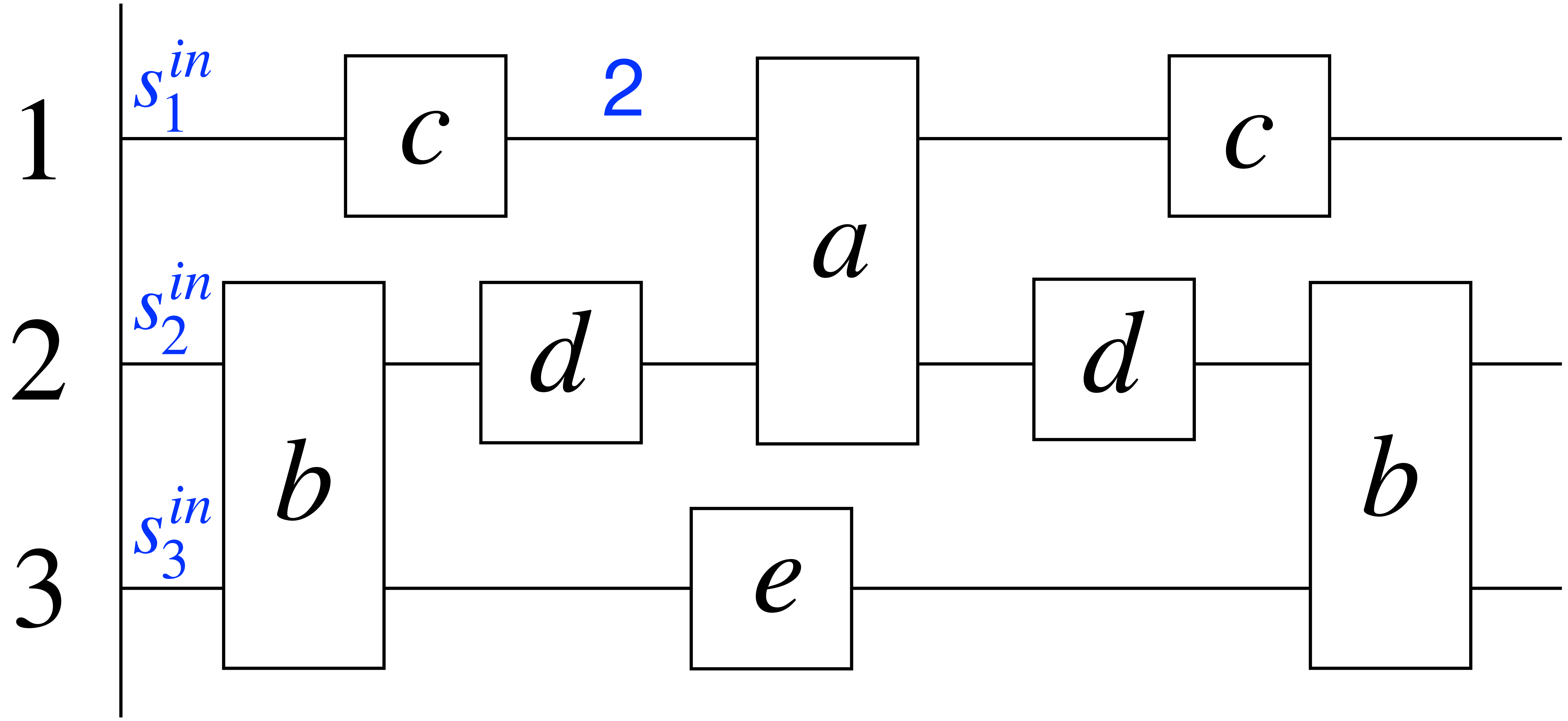

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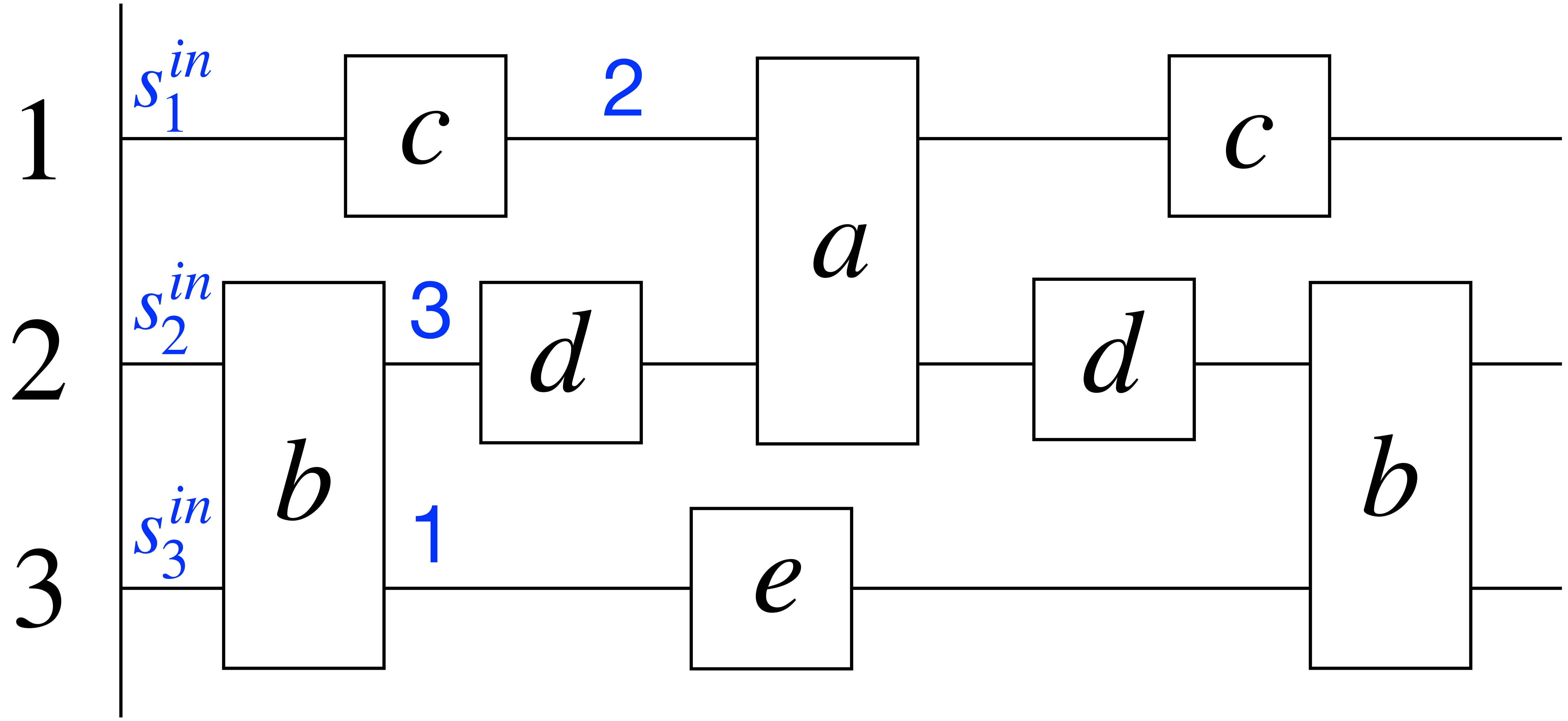

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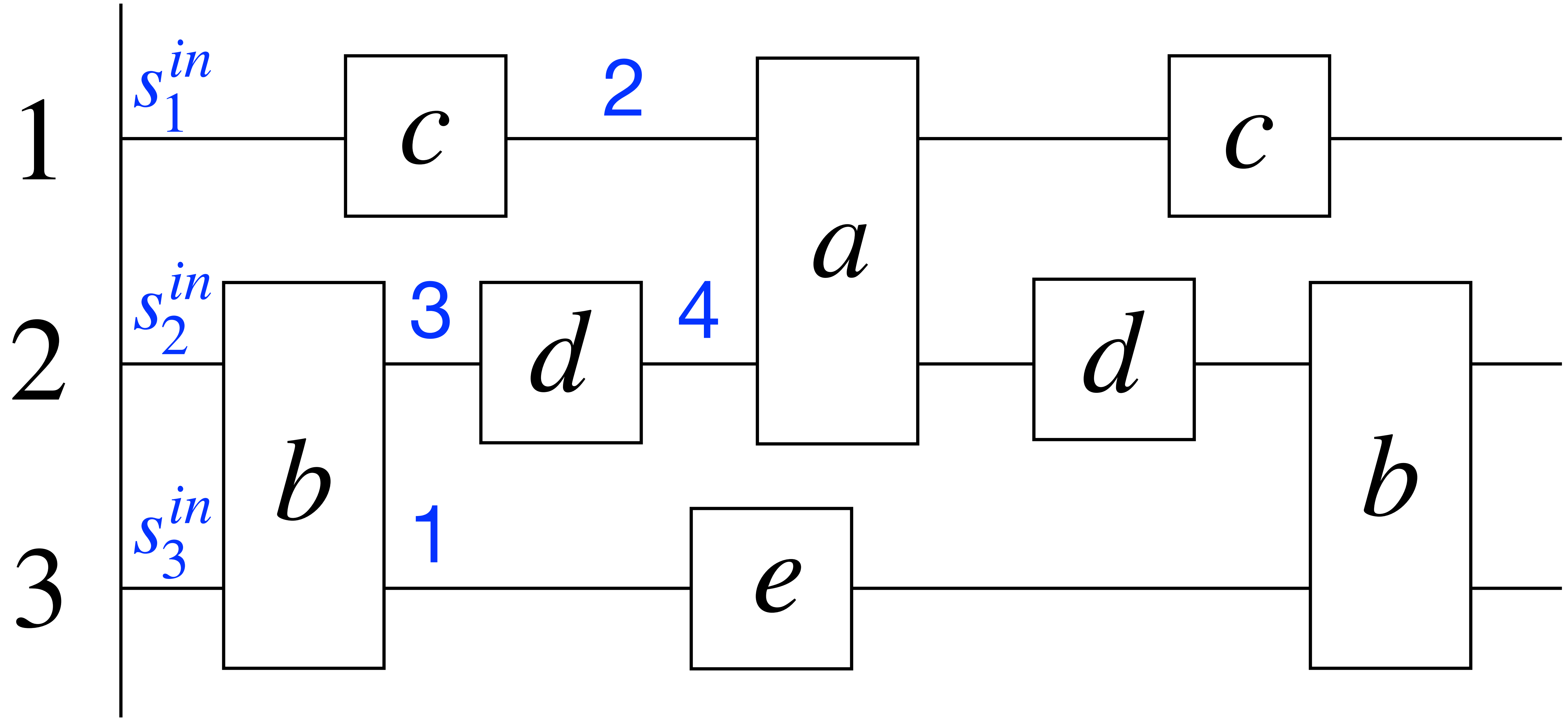

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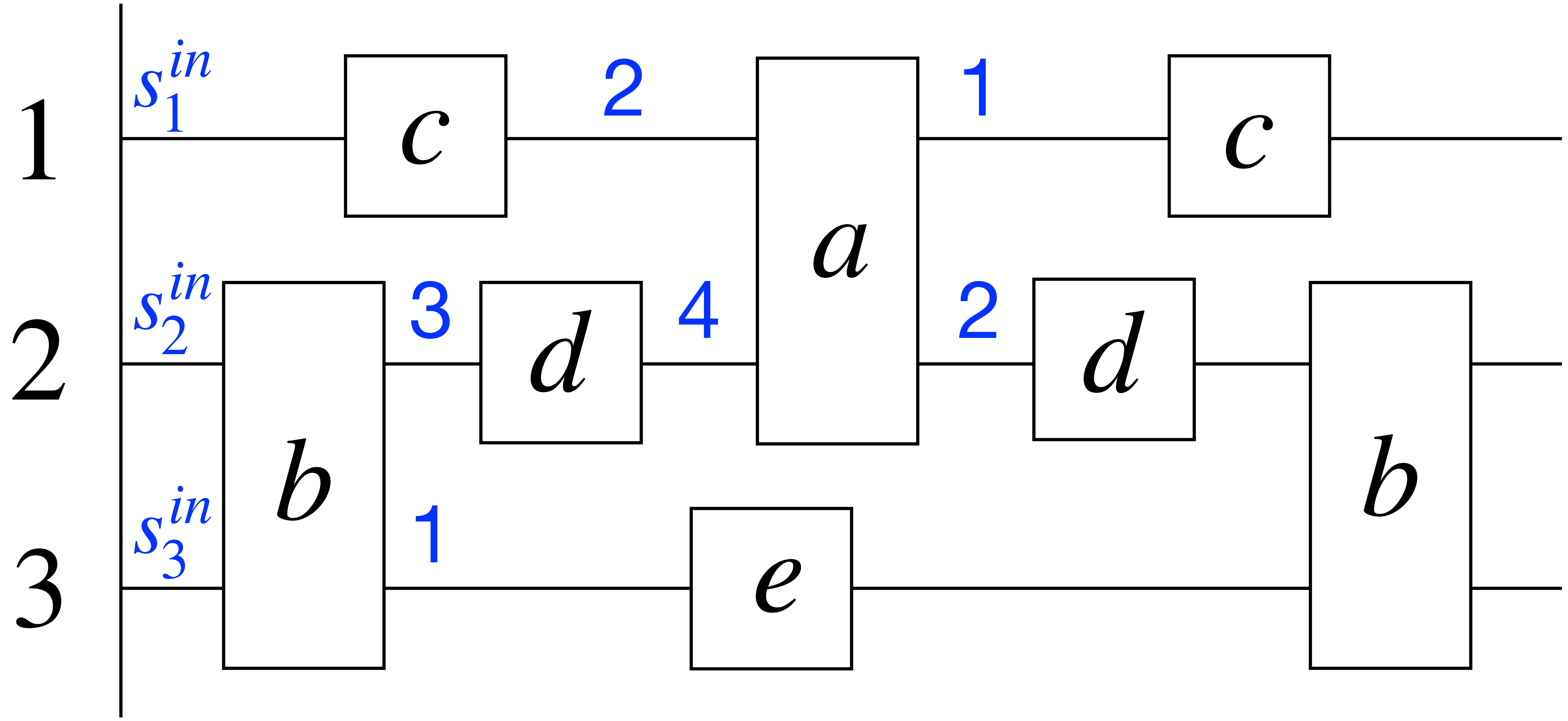

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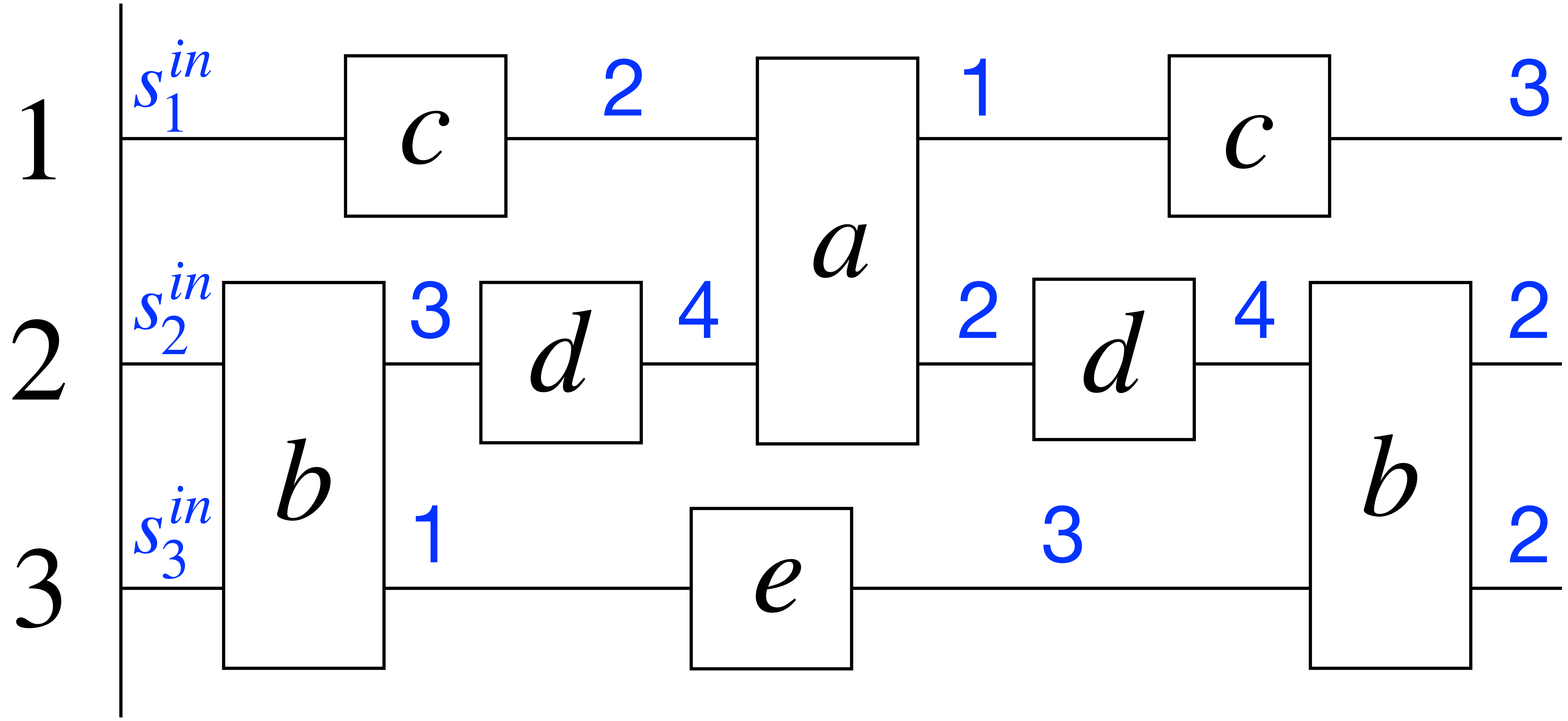

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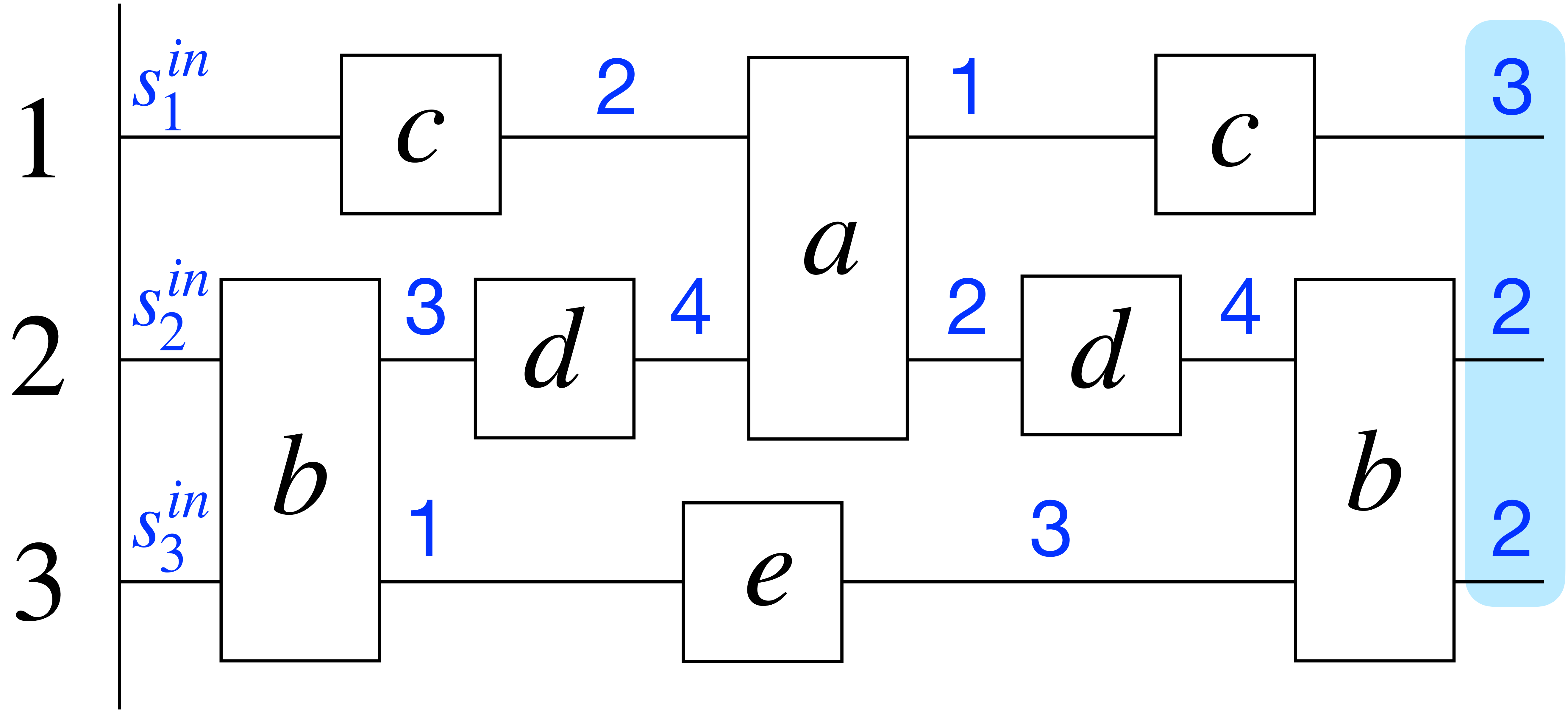

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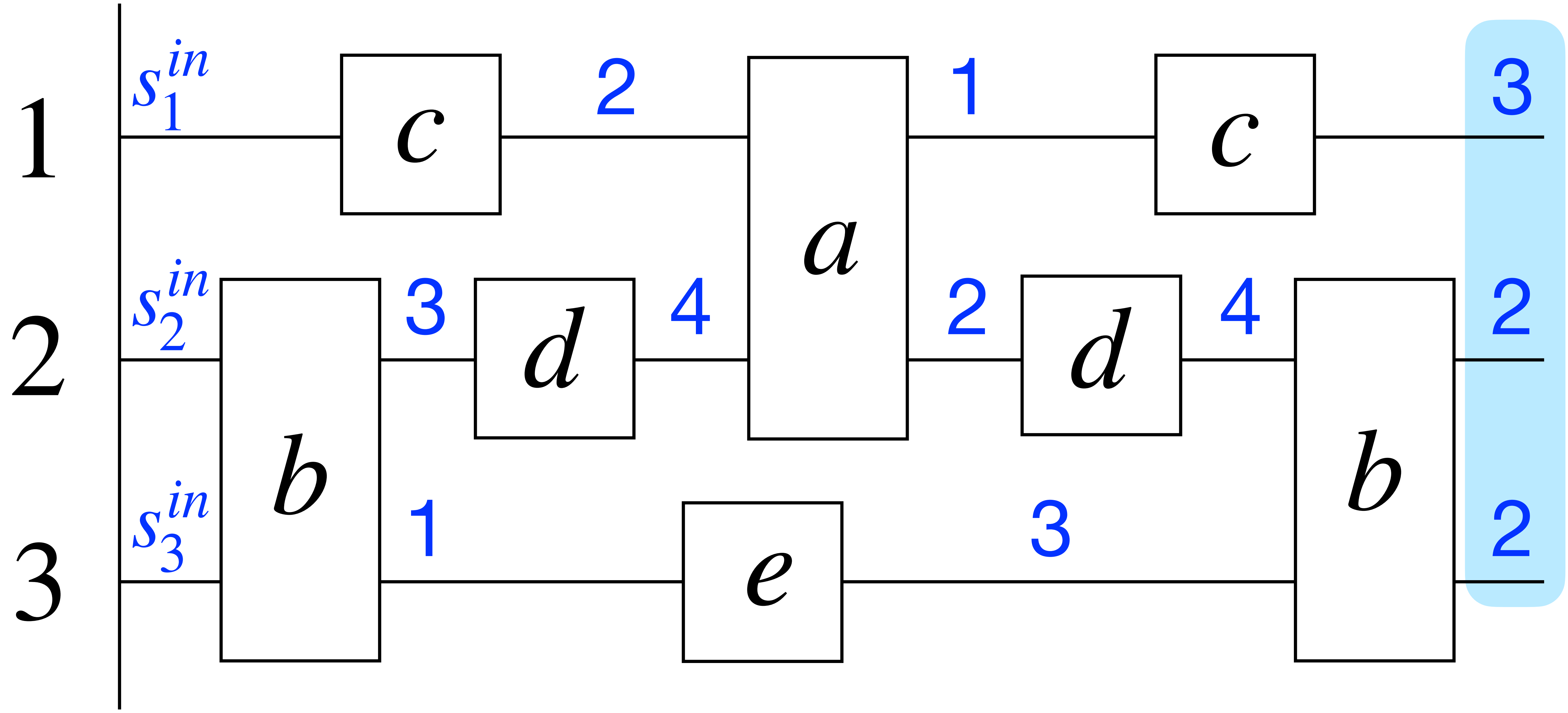

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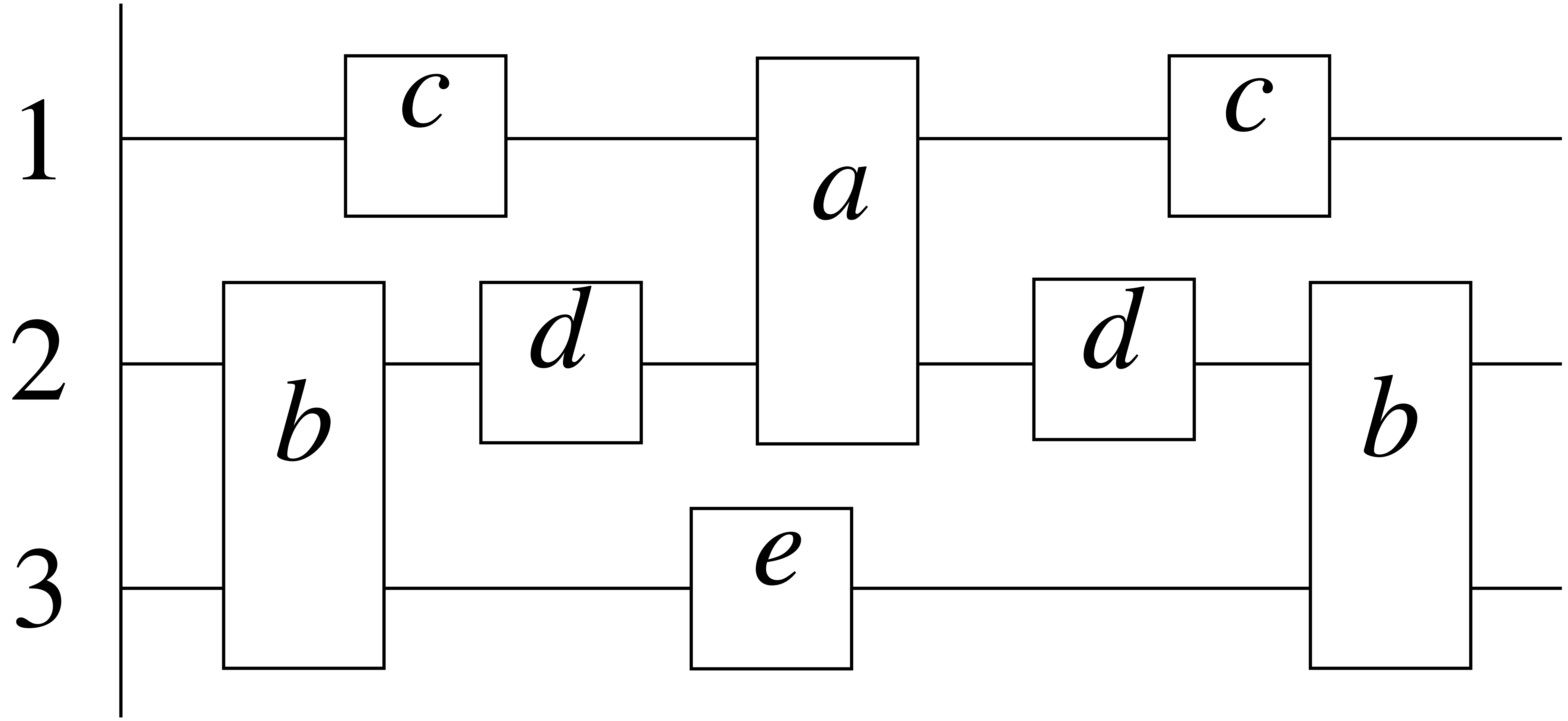
Theorem (Zielonka, 1987)

Asynchronous Automata = Regular Trace Languages

Labeling function

$\theta : Tr(\Sigma) \to Tr(\Sigma \times \Gamma)$



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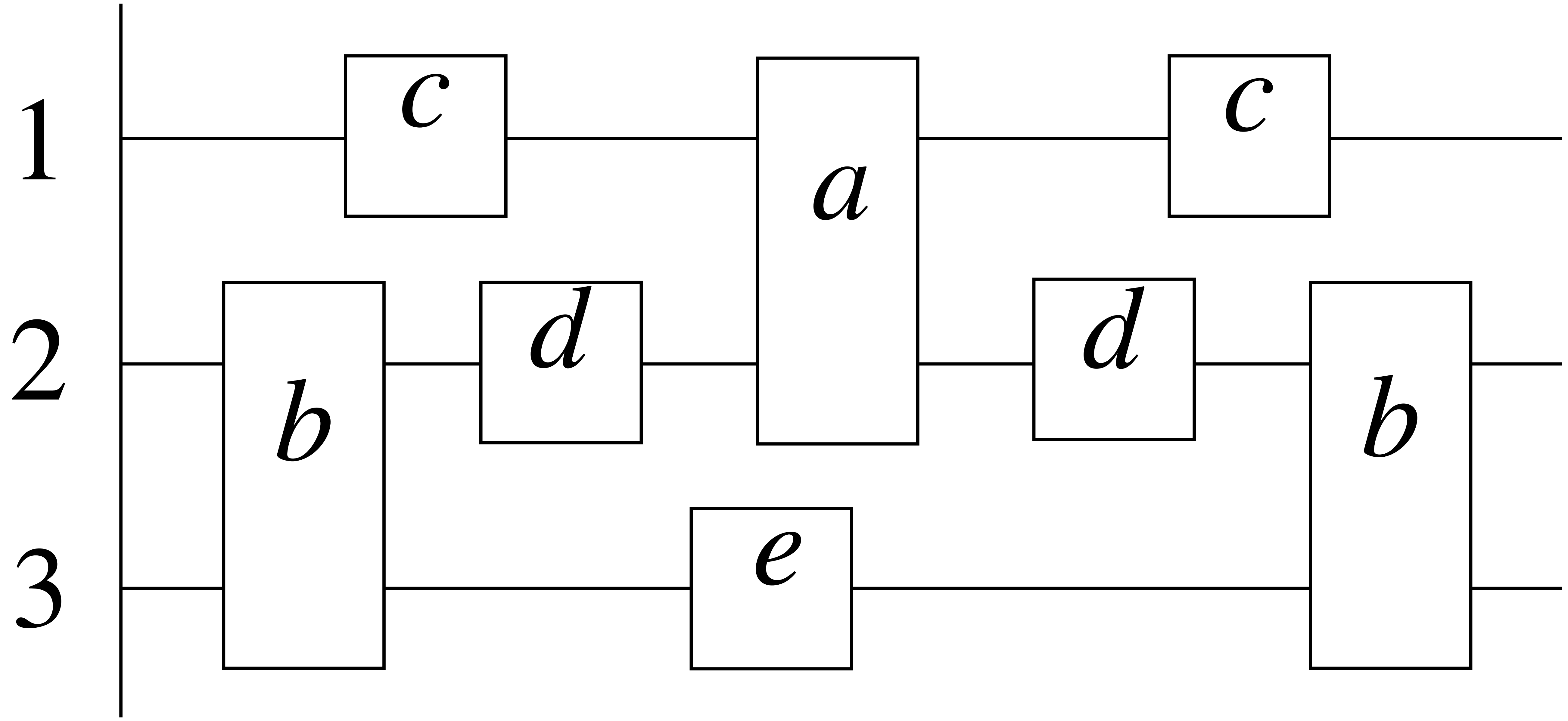
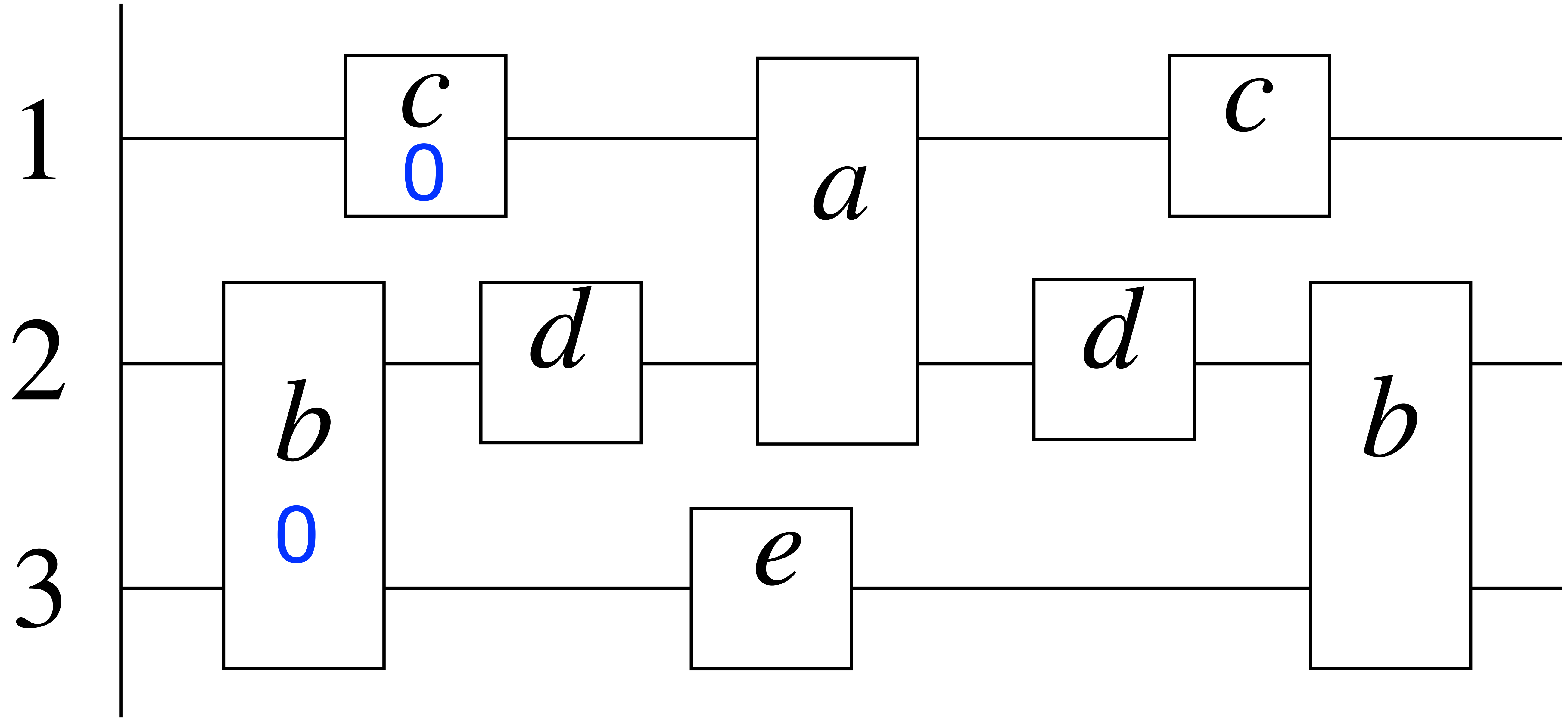


$\theta : Tr(\Sigma) \rightarrow Tr(\Sigma \times \Gamma)$

$\Gamma = \{0,1\}$ and $\mathrm{Y}_3 \leq \mathrm{Y}_2$

Labeling function



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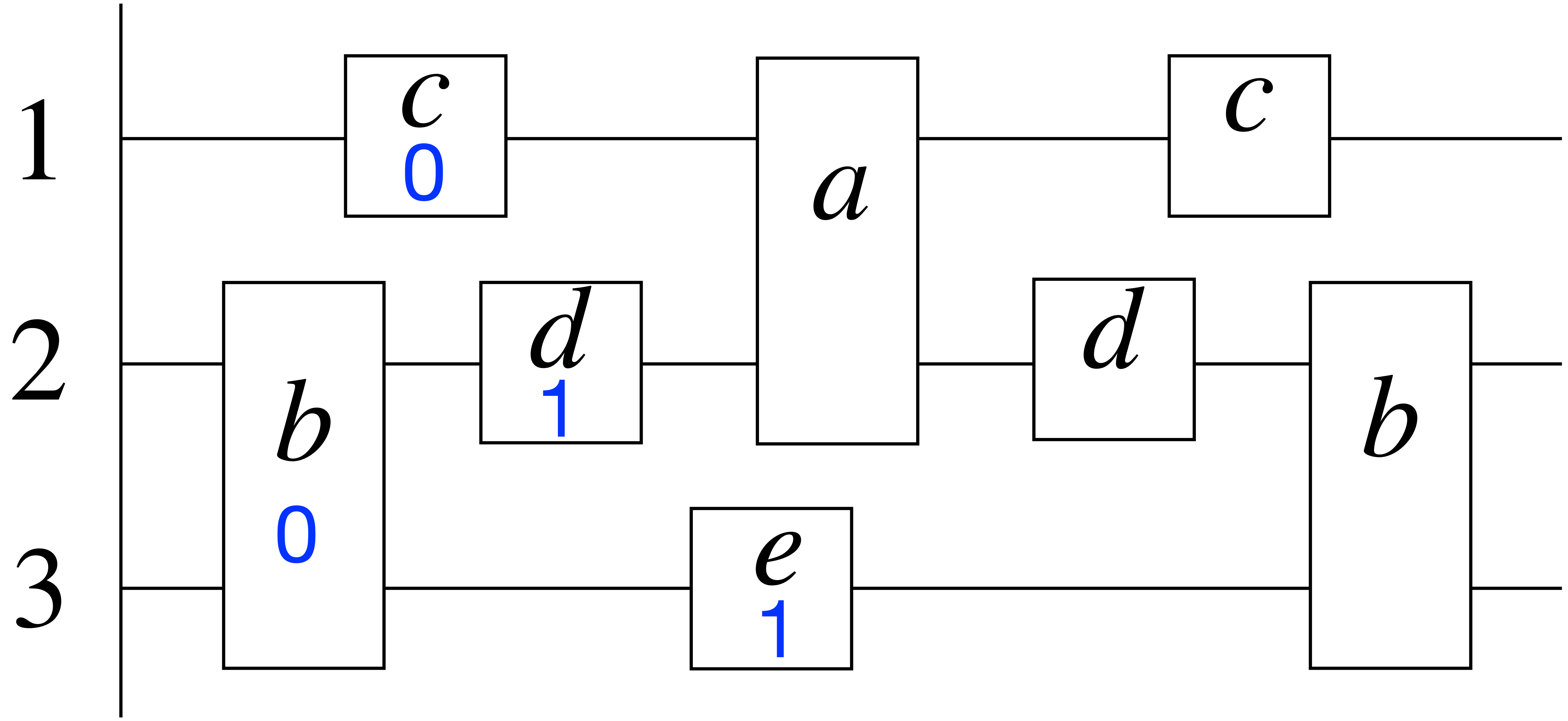

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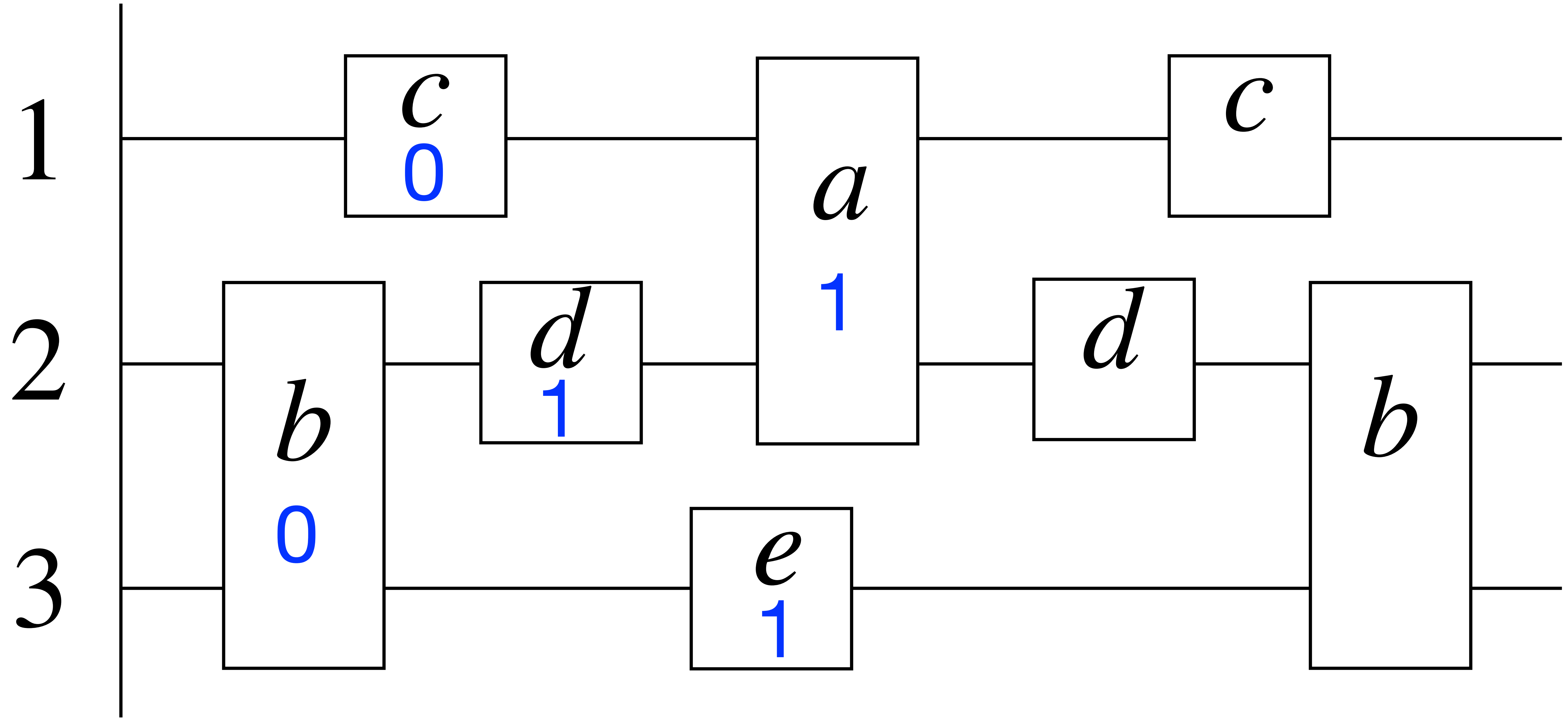

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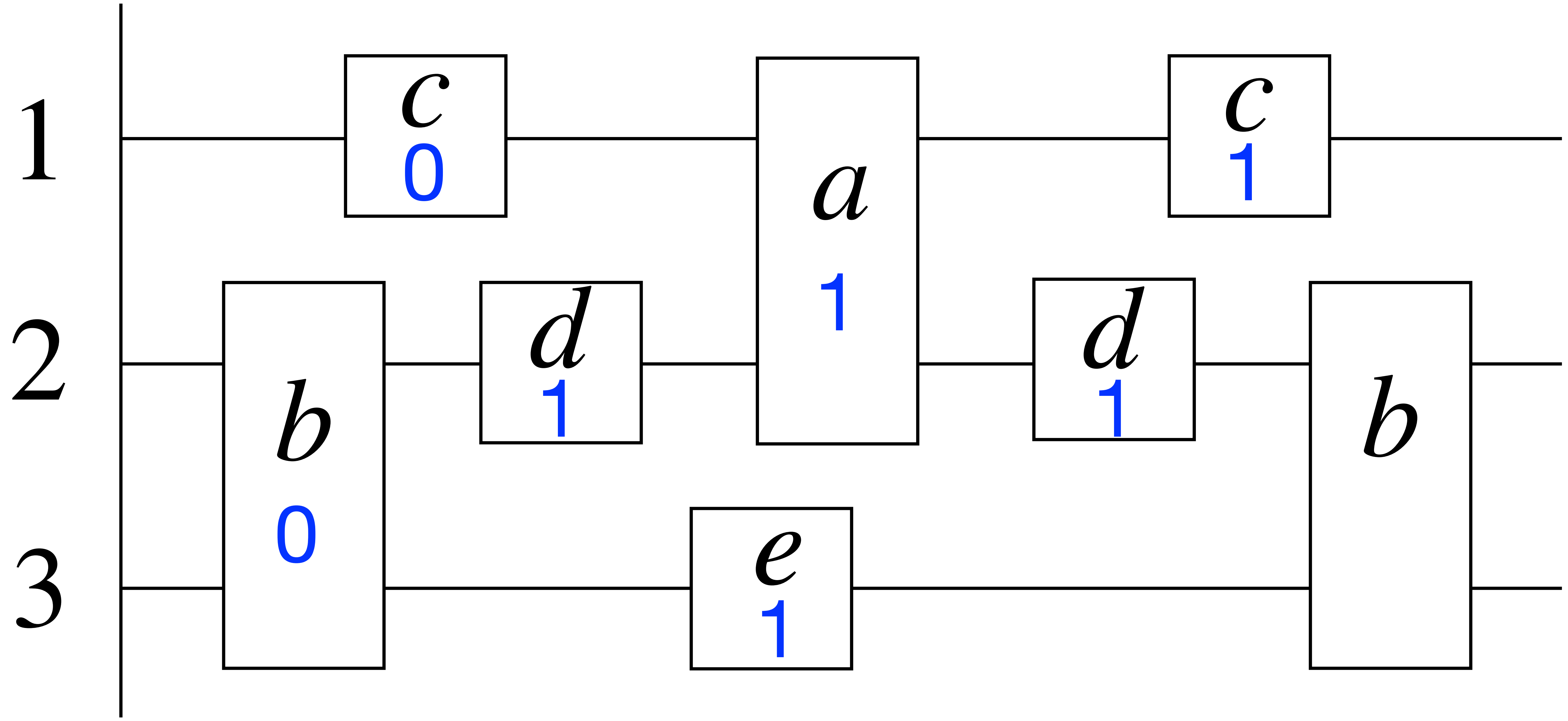

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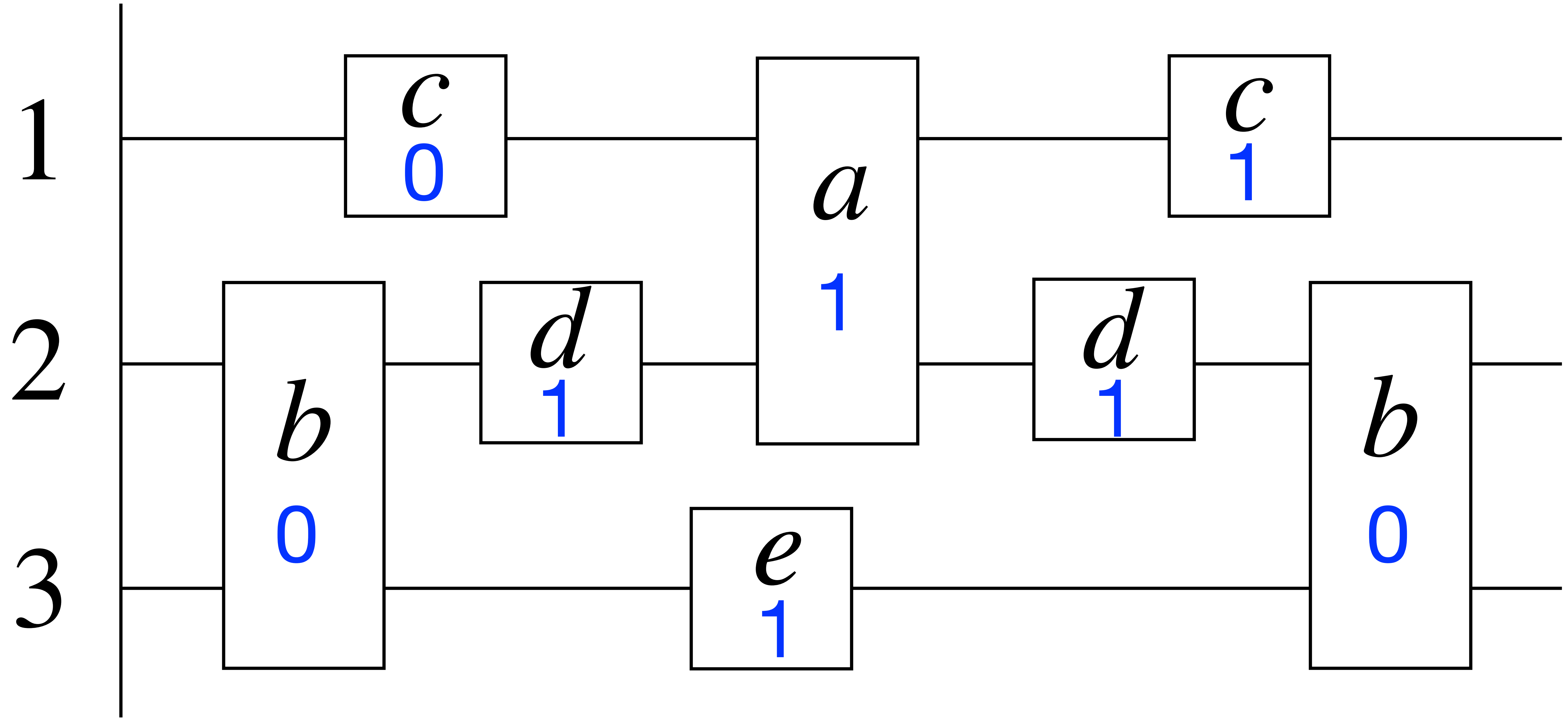

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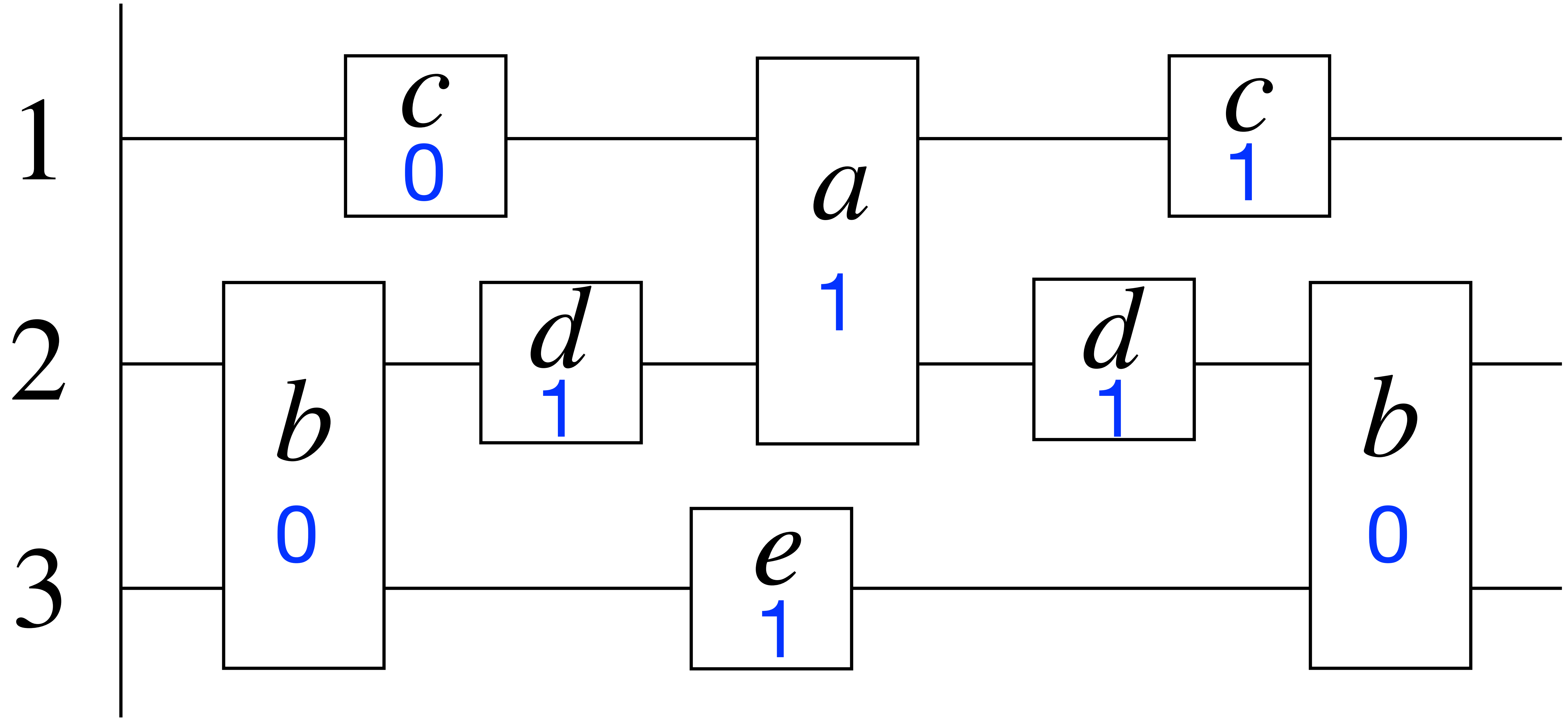

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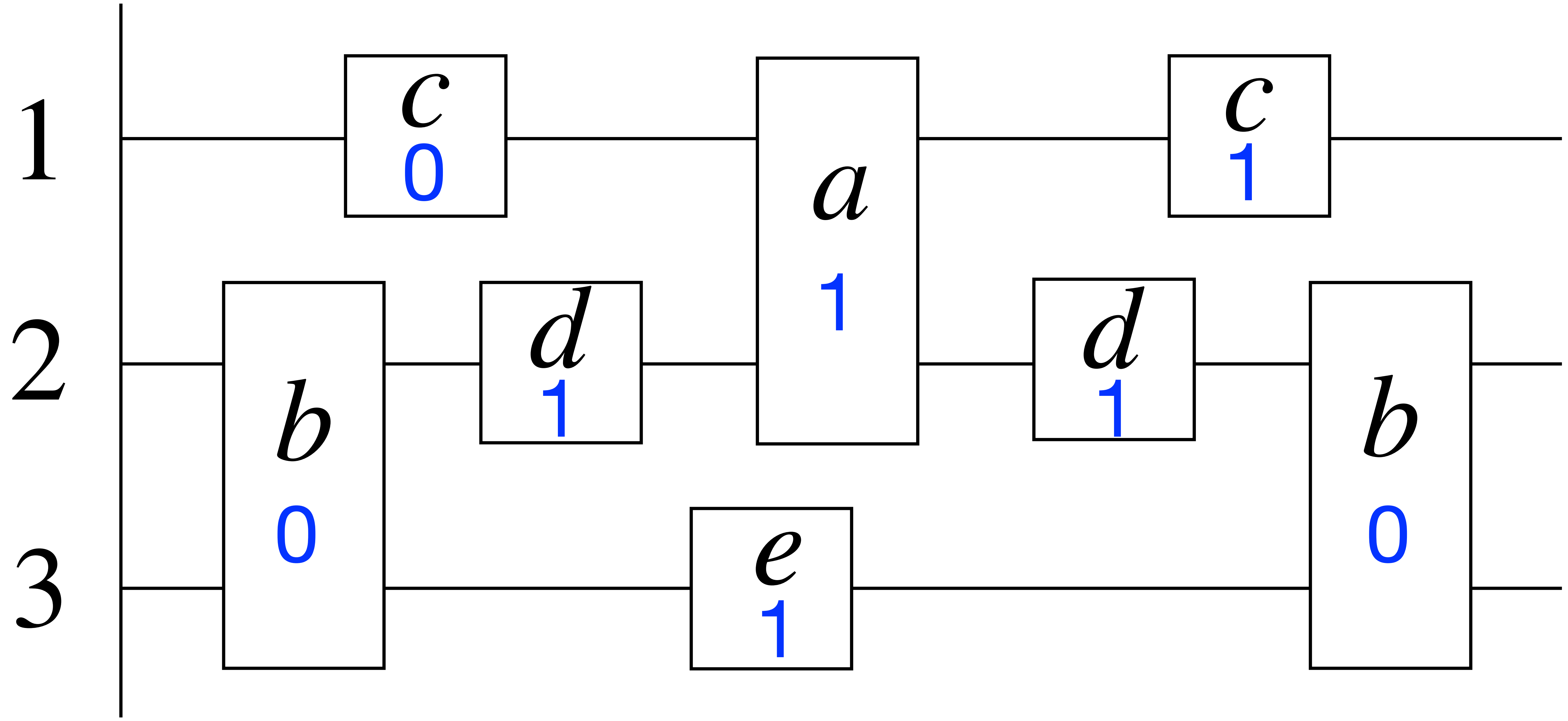
- $\mathscr{F}$ set of event formulas
- $\theta^{\mathscr{F}} : Tr(\Sigma) \to Tr\left(\Sigma \times \{0,1\}^{\mathscr{F}}\right)$

Asynchronous Labeling function



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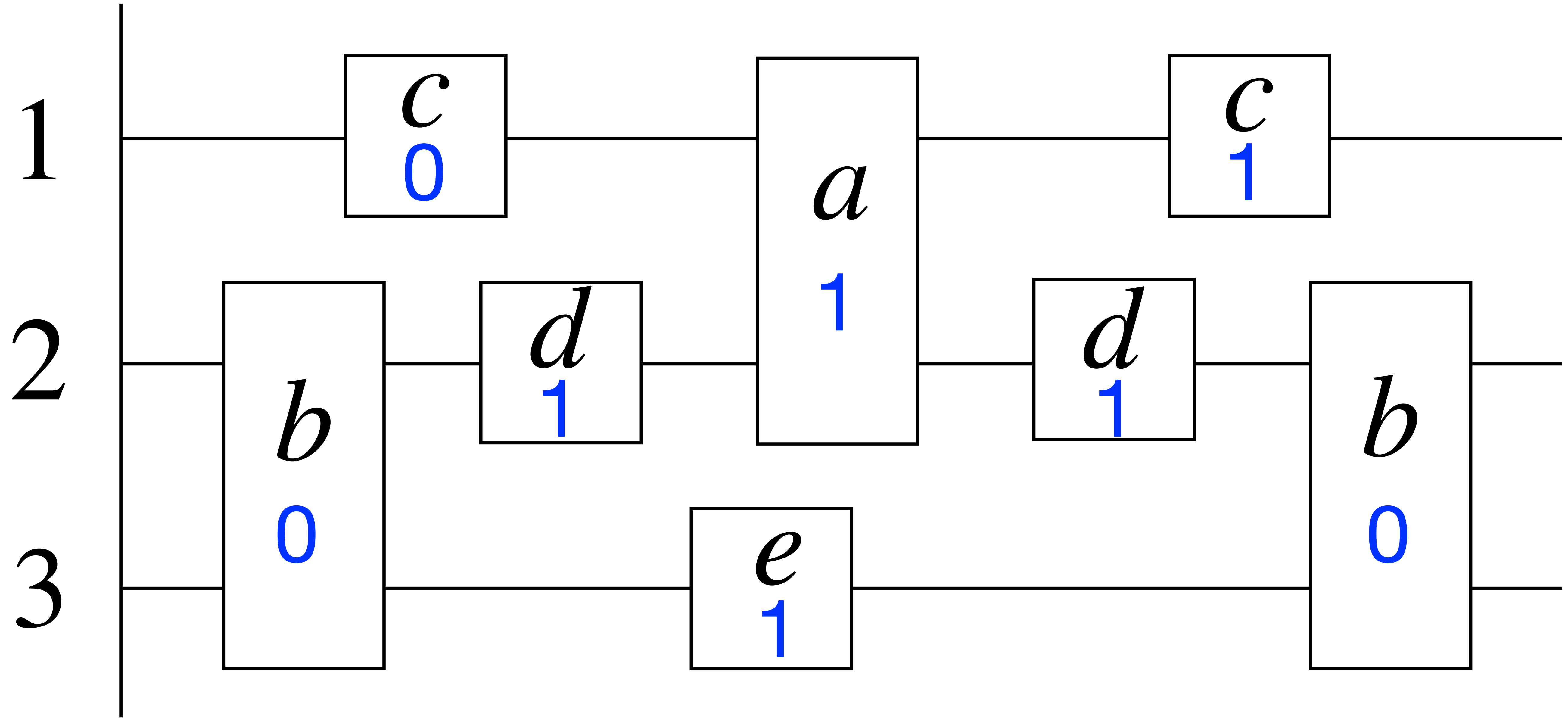

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Gossip (Asynchronous) Transducer

Theorem (Mukund-Sohoni 1997)

Let $\mathscr{Y} = \{\mathsf{Y}_i \leq \mathsf{Y}_j \,, \mathsf{Y}_{i,k} \leq \mathsf{Y}_j \;\mid\; i, j, k \in \mathscr{P}\}$.

There is an asynchronous letter-to-letter transducer $\mathscr{G}$ which computes

$$\theta^{\mathscr{Y}} : Tr(\Sigma) \to Tr\left(\Sigma \times \{0,1\}^{\mathscr{Y}}\right)$$

Outline

- ✅ Model of Concurrency: Mazurkiewicz Traces
- ✅ A propositional dynamic logic for traces
- ✅ Asynchronous automata (Zielonka) and asynchronous labeling functions
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Composition - Cascade product

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- Composition of labelling functions

$$Tr(\Sigma) \xrightarrow{\theta_1} Tr(\Sigma \times \Gamma) \xrightarrow{\theta_2} Tr(\Sigma \times \Pi)$$

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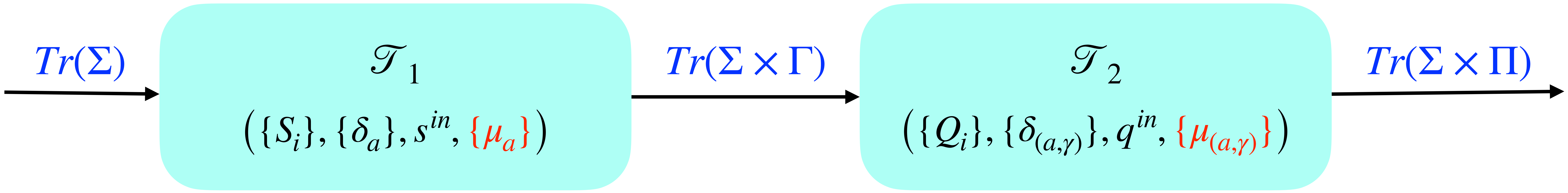

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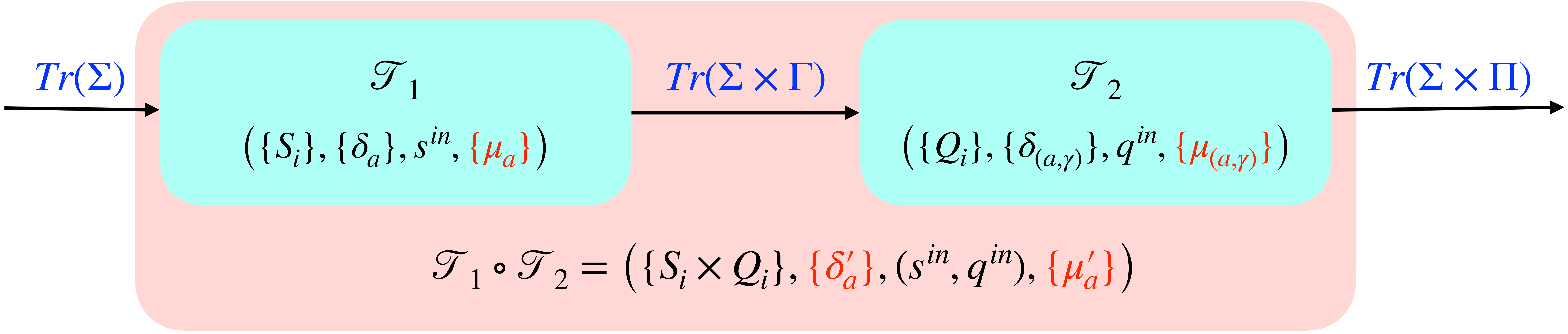

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$$\boxed{\delta'_a(s,q) = \left(\delta_a(s), \delta_{(a,\mu_a(s))}(q)\right)}$$

$$Tr(\Sigma) \longrightarrow \mathcal{T}_1 \; \left(\{S_i\}, \{\delta_a\}, s^{in}, \{\mu_a\}\right) \xrightarrow{Tr(\Sigma \times \Gamma)} \mathcal{T}_2 \; \left(\{Q_i\}, \{\delta_{(a,\gamma)}\}, q^{in}, \{\mu_{(a,\gamma)}\}\right) \longrightarrow Tr(\Sigma \times \Pi)$$

$$\mathcal{T}_1 \circ \mathcal{T}_2 = \left(\{S_i \times Q_i\}, \{\delta'_a\}, (s^{in}, q^{in}), \{\mu'_a\}\right)$$

Cascade Decomposition

Theorem 2

Any regular trace language is accepted by a cascade product of the gossip transducer followed by a sequence of local asynchronous transducers:

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- Regular = locPastPDL (Theorem 1)

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- For each event formula φ we construct by structural induction such a cascade product computing θ^{φ}

$$\varphi ::= a \mid \varphi \vee \varphi \mid \neg \varphi \mid \langle \pi \rangle \varphi \mid \mathsf{Y}_i \leq \mathsf{Y}_j \mid \mathsf{Y}_{i,k} \leq \mathsf{Y}_j$$

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  - Constants and boolean connectives are trivial
  - Local past Path expressions $\langle\pi\rangle\,\varphi$ are (rather) easy

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Corollary: Zielonka's theorem

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Each local asynchronous transducer $\mathscr{T}$ can be chosen to be (on its non-trivial component)

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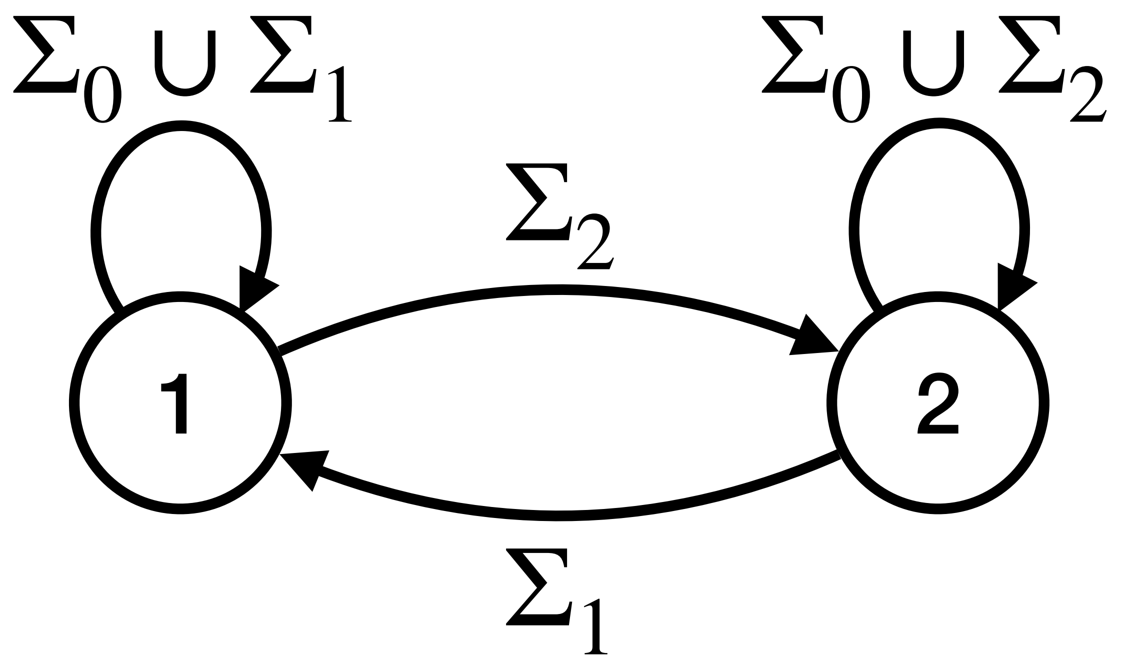

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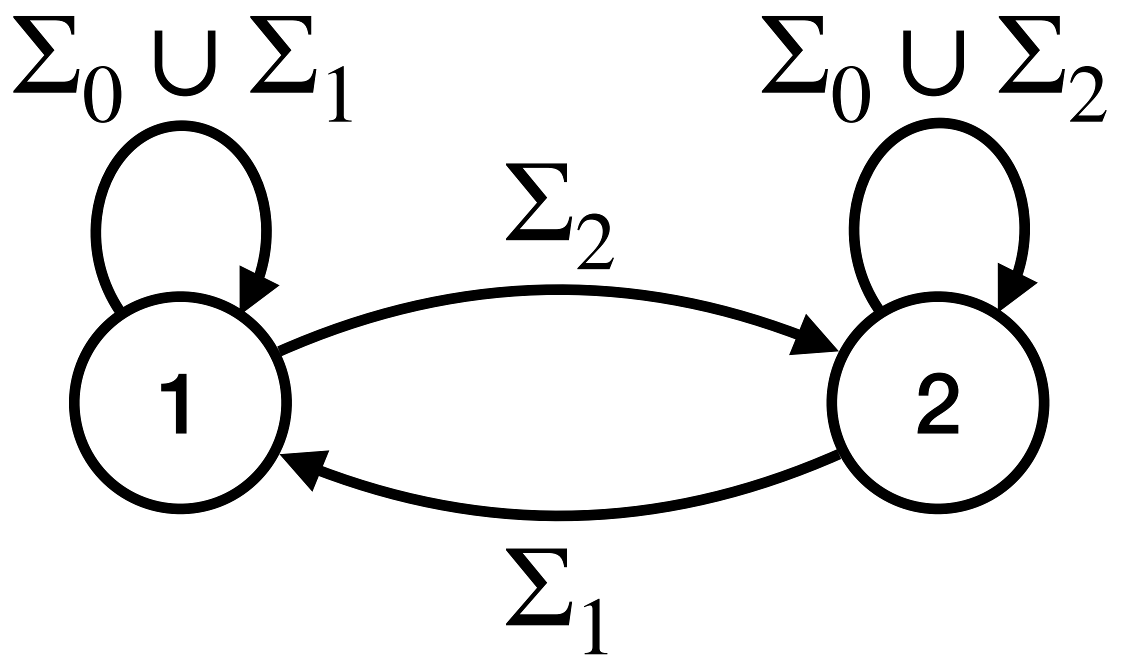
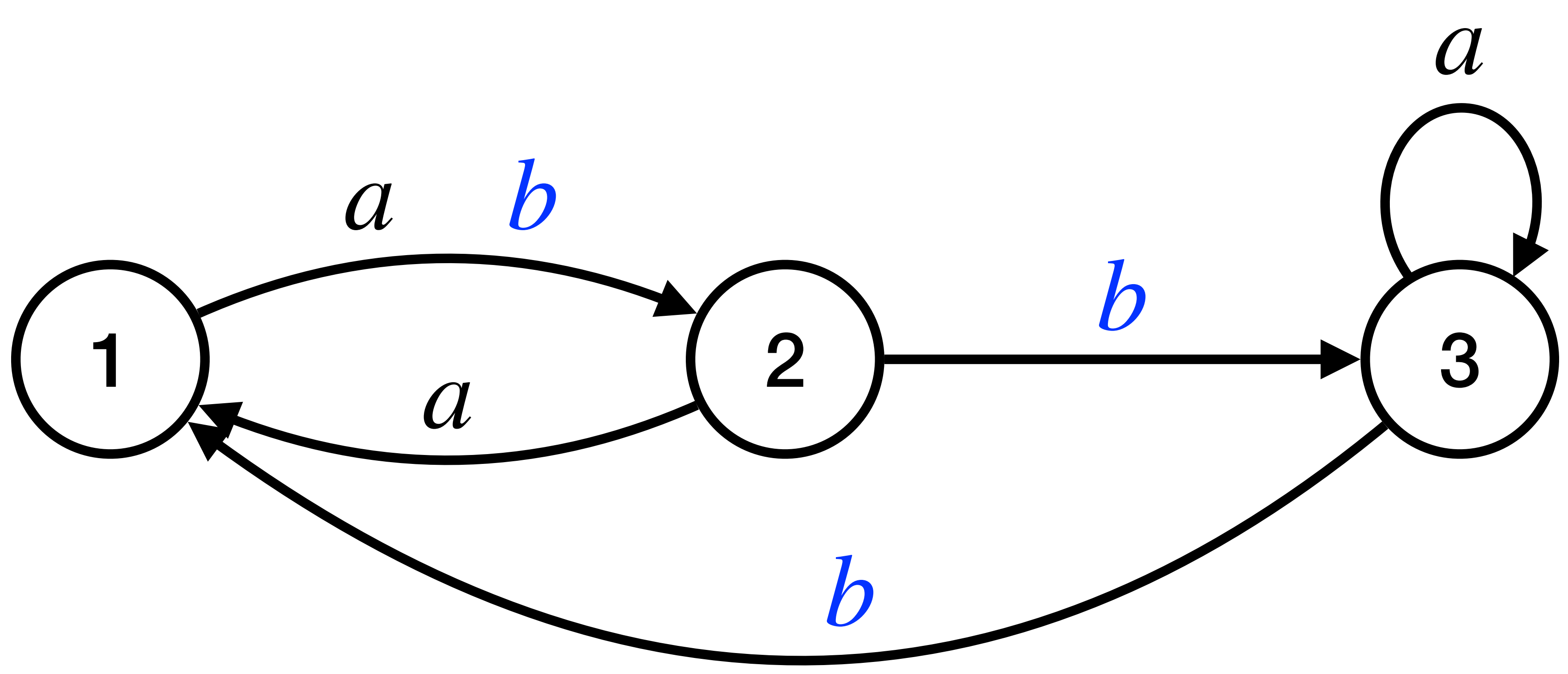


Permutation



Aperiodic = FO-definable

Theorem [Adsul, Gastin, Sarkar, Weil — Concur'20, LMCS'22]

Any aperiodic (FO) trace language is accepted by a cascade product of the gossip transducer followed by a sequence of local reset transducers:

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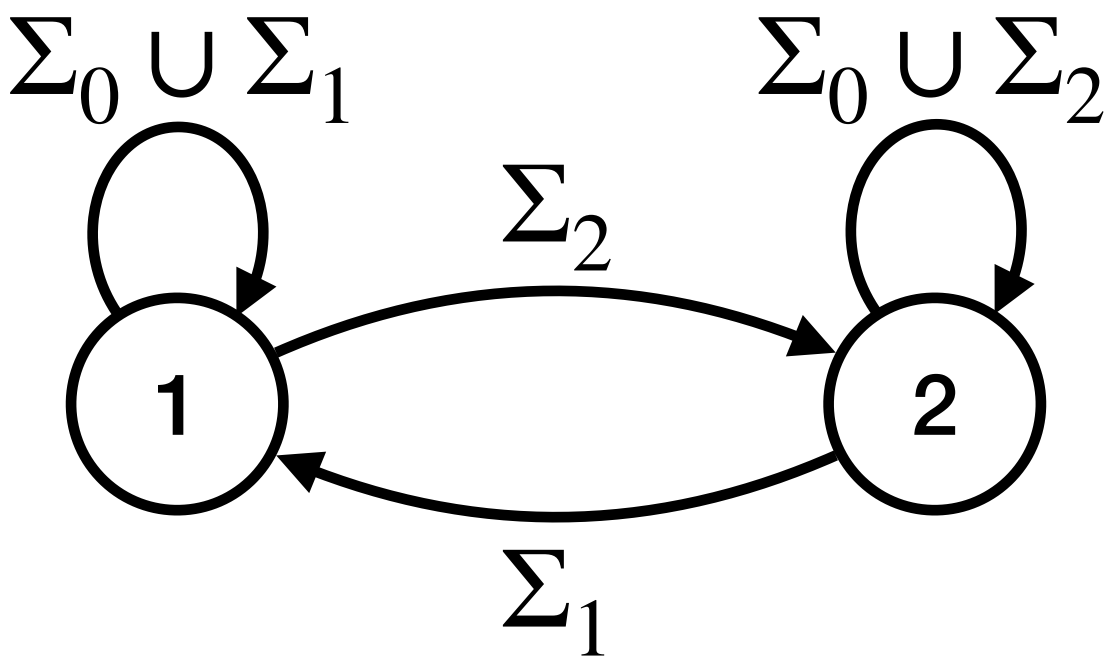

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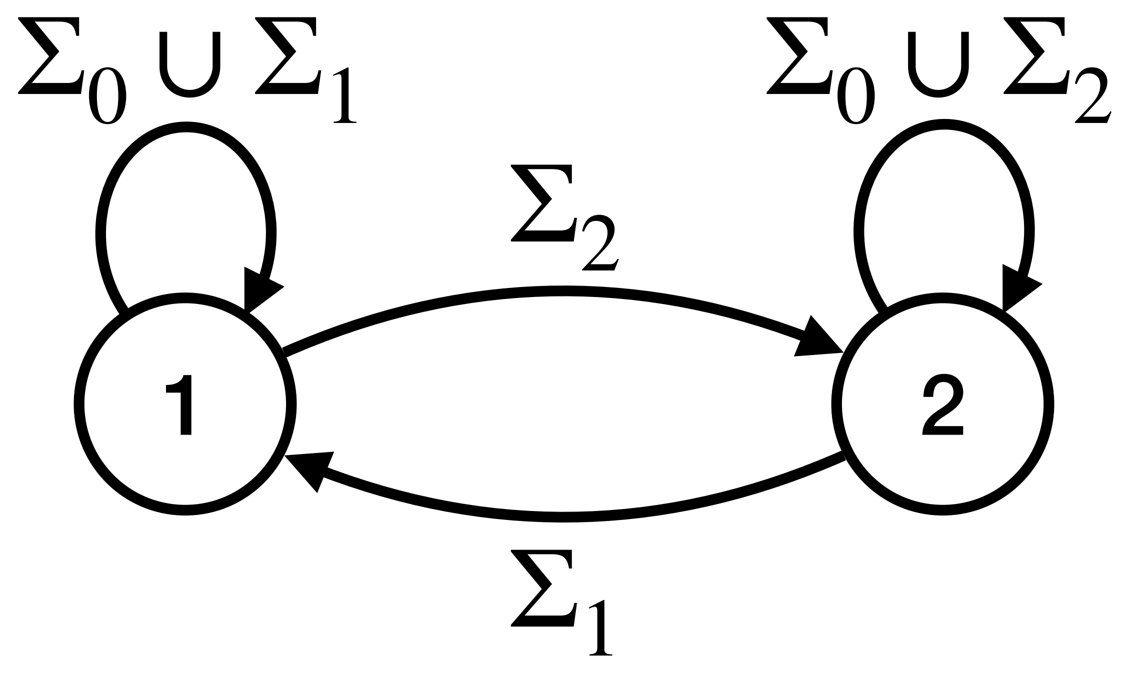
Direct proof (Not using Krohn-Rhodes theorem)

based on a past temporal logic $\mathrm{LTL}\left(\mathsf{Y}_i \leq \mathsf{Y}_j, \mathsf{S}_i\right)$ proved expressively complete for FO

Reset - $\mathscr{U}_2$



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Conclusion

Main results

- (Local) (past) Propositional dynamic logic expressively complete for regular languages
  - ▷ Specification language: natural, easy, expressive, good complexity
- Cascade decomposition using simple & local asynchronous automata/transducers
  - ▷ Allows inductive reasoning on automata

Open problems

- Expressivity of locPastPDL without some constants $\mathsf{Y}_{i,k} \leq \mathsf{Y}_j$, $\mathsf{Y}_i \leq \mathsf{Y}_j$, $\mathsf{L}_{i,k} \leq \mathsf{L}_j$, $\mathsf{L}_i \leq \mathsf{L}_j$?
- Can we get rid of the gossip transducer in the cascade decomposition?
  - ▷ Yes if the architecture (communication graph) is acyclic! (CONCUR'20, LMCS'22)
- Can we decompose the gossip transducer in a cascade of local asynchronous transducers?
- Generalisation to Message Sequence Charts (Message passing automata)?

Thank you for your attention!