ALGEBRAIC STRUCTURE OF FLOWS OF A REGULAR COLOURED NET

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ABSTRACT

This paper introduces a new flow calculation theory for a wide subclass of coloured nets : the regular nets (R.N.). Their parametrization allows to study at the same time the flows for all nets differing only by the sizes of the colour sets. The algebraic structure of the flows subspace provides a fundamental decomposition theorem leading to an algorithm computing a flow basis for a parametrized regular net. The modelling of a significant classical example is presented with the computation of a basis of flows.

INTRODUCTION

Recently, some extensions of Petri nets - coloured nets [Jen82] and predicate transition nets [Gen81] - have been introduced to model in a concise way, classes of objects with a similar behaviour (i.e. file's readers, input-output buffers,…) for protocols or distributed applications. In fact a coloured net is equivalent to a Petri net, but in the coloured net one needs only one subnet to represent similar objects behaviours instead of one subnet per object since objects identities are kept by means of colours.

Coloured nets may be hardly studied by unfolding them and by applying classical means - accessibility graph, flows calculus, reduction … [Bra83] - to the equivalent Petri nets. But this method would have severe drawbacks : the size of the unfolded nets may be very large, the unfolding needs to fix the system parameters (number of objects i.e. colour ranges) and moreover the results cannot in general be folded and interpreted in the original coloured nets.

So many researchers have tried to extend the main results of the Petri net theory and in particular the flows calculus by the Gauss elimination. The first heuristic method [Jen81] uses a set of transformation rules to reduce the incidence matrix of a coloured net without changing the set of flows. Afterwards Vautherin and Memmi [Vau84] compute directly a family of flows for an unary predicate transition net by applying Gauss elimination on a module generated by the constants and variables appearing in the net. However this family is generally not a basis of flows. In [All84],[Sil85] one applies a directed Gauss elimination in order to find folded flows weighted by the colour functions and their inverses but this method needs a condition on the incidence matrix and moreover one must fix the cardinalities of the colour sets. Other researches [Gen82] describe different kinds of flows but without giving any method to find them.

In fact these researches have failed to bring a complete and adequate answer to the flows calculus problem since the predicate transition nets allow too much arbitrary relations between arc variables [Lau85]. Hence some restrictions are needed to obtain regularity and to simplify the algebraic structure of the flows space by decomposition in outstanding subspaces.

Here we define the regular nets that are a subclass of coloured nets. In these nets the colour functions must be in a normal form depending on colour domains. Moreover to study at the same time all the nets having identical structure but different cardinalities of the colour ranges, we have parametrized our definition. The parametrized regular nets include the ones defined in the Memmi's thesis [Mem83] and allow to model complex distributed systems.

To study the flows space of a regular net, we build Petri nets deduced from the regular net, such as the underlying net [Jen82] and mainly the synchronized nets which measure the difference between tokens of two colours of the same domain. Flows of these Petri nets are outstanding flows of the original regular net and moreover we prove a fundamental theorem of unique decomposition of the flows space over the subspace of the flows of the underlying net and subspaces of the flows of the synchronized nets. These last flows represent synchronizations between colours and were not defined till now. An algorithm is given to compute a basis of flows for a parametrized regular net and immediately obtain the bases for the whole family of equivalent nets. This complete result is the main advantage of our theory.

General notations

- $\exists!$ means "there is one and only one…"
- **N** is the set of non negative integers
- **Z** is the set of integers
- **Q** is the set of rational numbers
- M.N , where M and N are matrices, denotes the matrices product (this notation includes the product of a vector by a matrix, since a vector is a special case of matrix)
- ${}^tM$, where M is a matrix n x p, denotes the matrix p x n such that : ${}^tM_{i,j} = M_{j,i}$
- Let U be a finite set , then the set of functions from U to **N** is denoted Bag(U). An item a of Bag(U) is noted $\sum a_u.u$ where the summation is over $u \in U$.
- A partial order on Bag(U) is defined by : $a \geq b$ if and only if $\forall\, u \in U$, $a_u \geq b_u$
- The sum of two items of Bag(U) is defined by $a+b = \sum (a_u+b_u).u$
- The difference between two items $a \geq b$ of Bag(U) is defined by : $a-b = \sum (a_u-b_u).u$

I REGULAR NETS AND PARAMETRIZED NETS

I.1 Coloured nets

We recall the definitions of a coloured net, the firing rule in a coloured net, the incidence matrix and the flows of a coloured net. Since the regular nets are a subclass of coloured nets, the definition of a regular net is a restriction of the first definition and the other definitions can be applied without any transformation.

Definition 1 A coloured net R = <P,T,C,I^+,I^-,M> is defined by :

- P the set of places
- T the set of transitions
- C the colour function from P U T to Ω , where Ω is the set of finite and not empty sets. An item of C(s) is called a colour of s and C(s) is called the colour set of s.
- I^+ (I^-) is the forward (backward) incidence matrix of P x T , where $I^+(p,t)$ is a function from C(p) x C(t) to **N** (i.e. a function from C(t) to Bag(C(p)))
- M the initial marking of the net is a vector of P, where M(p) is a function from C(p) to **N** (i.e. an item of Bag(C(p)))

Notation We note $I^+,I^-(p,t,ct)$, where ct belongs to C(t), the corresponding item of Bag(C(p)).

Definition 2 The firing rule is defined by :

- A transition t is enabled for a marking M and a colour ct $\in$ C(t) if and only if : $\forall$ p $\in$ P , $M(p) \geq I^-(p,t,ct)$
- The firing of t for marking M and a colour ct $\in$ C(t) gives a new marking M' defined by : $\forall$ p $\in$ P , $M'(p) = M(p) - I^-(p,t,ct) + I^+(p,t,ct)$

Definition 3 The incidence matrix I of a coloured net is defined by :

- $I = I^+ - I^-$, then I(p,t) is a function from C(p) x C(t) to **Z**
- I can be also wieved like a matrix of U (p,c) x U (t,c') of integers where the first union is over p $\in$ P and c $\in$ C(p) and the second union is over t $\in$ T and c $\in$ C(t), by the simple transformation : I((p,c),(t,c')) = I(p,t)(c,c')

Definition 4 The vector space of coloured places E of a coloured net is defined as the rational vector space on U (p,c) where the union is over p $\in$ P and c $\in$ C(p).

Then a vector of E can be written $v = (v_{p,c})$ with $v_{p,c} \in$ **Q**

or $v = \Sigma\, v_{p,c} \,.\, (p,c)$

Definition 5 The set of flows E' of a coloured net is a subset of E defined by :

$E' = \{ v \in E \,/\, {}^{t}I.v = 0 \}$

In fact, because of the linearity of the product of matrices, E' is a vector subspace of E .

Remark The definition of flows in [Jen82] involves a weight function for each place p, from C(p) to Bag(U) where U is a finite set. In our case the weight function is defined from C(p) to **Q**. But the first definition can be reduced to the second in the following way :

Let $\Sigma\, f_p.p$ be a flow for the first definition, then this flow is equivalent to the family of flows $\Sigma\, f_{p,u}.p$ where u belongs to U and where $f_{p,u} = Proj_u(f_p)$ is defined from C(p) to **N** (hence from C(p) to **Q**)

Definition 6 Let F be a subspace of E, F' is the subspace of the flows of F defined by : $F' = \{ v \in F \,/\, {}^{t}I.v = 0 \}$

I.2 Regular nets

Since the regular nets are a subclass of coloured nets, they are defined by restriction. There are two kinds of restrictions :

- On the colour sets
- On the items of I^+ and I^-

a. The colour sets of a regular net

Given two places of the net, then either their colour sets are identical either their colour sets have an empty intersection :

$\forall\, p,p' \in P$, $C(p) = C(p')$ OR $C(p) \cap C(p') = \emptyset$

Then we call the different colour sets of places $\{C1,\ldots,Ck\}$, the classes of the net. These classes have two properties :

$\forall\, i \neq j$, $Ci \cap Cj = \emptyset$ AND $\forall\, p \in P$ $\exists\, i$ such that $C(p) = Ci$

Ci is called the class of p

All the transitions have the same colour set $C(t) = C1 \times \ldots \times Ck$, that means a firing of a transition t selects exactly one colour per domain. As we shall see in the following paragraph, if no place with colour set Ci is connected to t the firing of t is independant of the colour of Ci distinguished by the firing.

b. The incidence matrices of a regular net

Let $p \in P$ such that $C(p)=Ci$ and $t \in T$, then $I^+,I^-(p,t)$ has the following form :

$a.Si + b.Xi : Ci \times (C1 \times \ldots \times Ck) \longrightarrow \mathbf{N}$, with $a,a+b \in \mathbf{N}$ where :

$a.Si + b.Xi(c,c1,\ldots,ck) =$ **If** $c=ci$ **Then** $a+b$ **Else** a

Using the bag notation :

$a.Si + b.Xi : (C1 \times \ldots \times Ck) \longrightarrow Bag(Ci)$, with $a,a+b \in \mathbf{N}$ where :

$a.Si + b.Xi(c1,\ldots,ck) = \sum a.c + (a+b).ci$

where the summation is over $c \in Ci$, $c \neq ci$

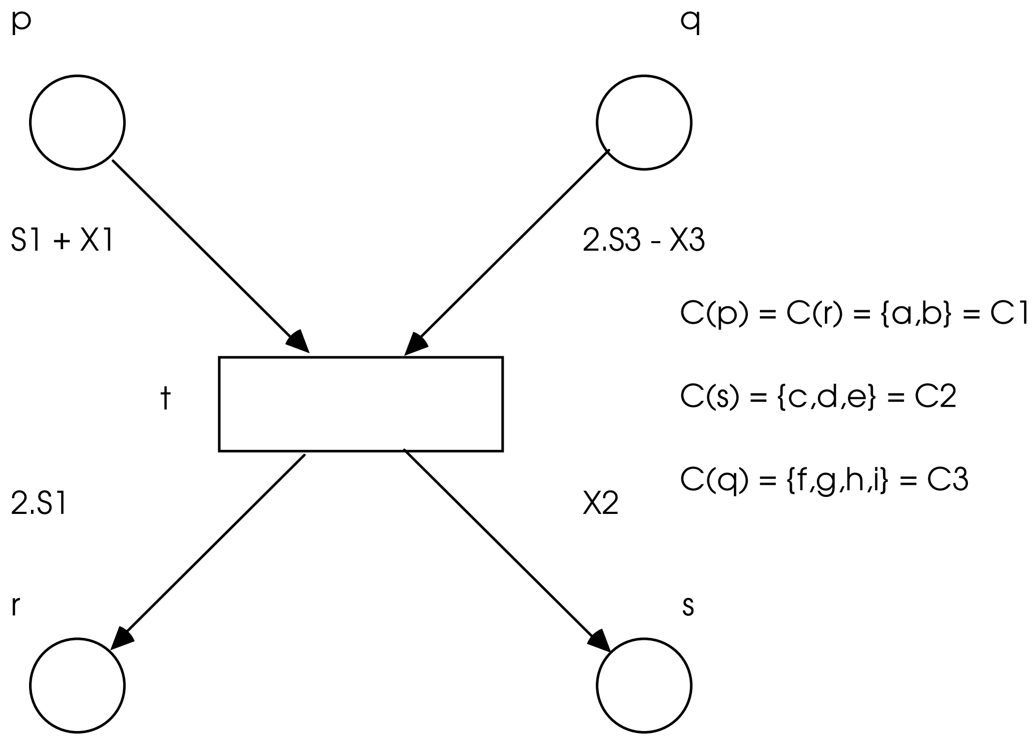
Interpretation

- An arc from p to t the valuation of which is Xi, means that to fire t one needs a token coloured by the colour of Ci selected by the firing of t in p.
- An arc from p to t the valuation of which is Si, means that to fire t one needs a token per colour of Ci in p.
- An arc from p to t the valuation of which is a.Si+b.Xi, means that to fire t one needs "a+b" tokens coloured by the colour of Ci selected by the firing of t and "a" tokens per colour different from the colour of Ci selected by the firing of t in p.

Definition A regular net is a coloured net which verifies the restrictions defined in **a** and **b**.

The definitions 2,3,4,5,6 about the coloured nets can be immediatly applied to the regular nets. We give now an example of a regular net and a firing in this net.

Example



Let M be a marking defined by (using the bag notation) :

- M(p) = 2.a + 2.b
- M(q) = 3.f + 2.g + 2.h + i
- M(r) = b
- M(s) = 0

Then t is enabled for M and for the colour (a,d,i) and the new marking is obtained by :

- M'(p) = b
- M'(q) = f
- M'(r) = 2.a + 3.b
- M'(s) = d

I.3 **Parametrized regular nets**

In a coloured net, the colour sets are generally not defined by enumeration (set of readers, set of buffers,…). So the numbers of items of a colour set can be supposed variable and it is always interesting to obtain properties independantly of these numbers. We call these numbers the parameters of the net and a parametrized regular net (P.R.N.) is a regular net where the cardinalities of Ci are let variable.

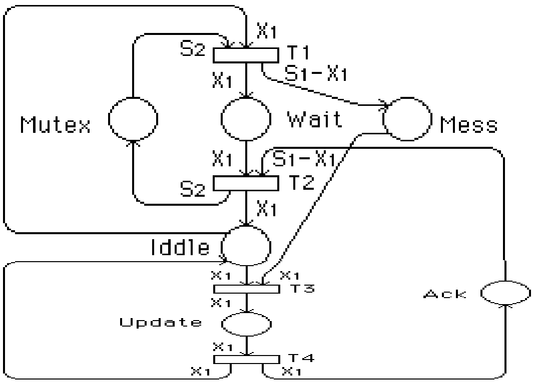
We note :

- ni the fixed cardinality of Ci in a R.N.
- Ni the variable cardinality of Ci in a P.R.N. In this case, {N1,…,Nk} are called the parameters of the net.

I.4 Example of modelling by P.R.N.

We present the modelling of a data base management with multiple copies [Jen81]. To modify the data base, a site must get a grant modelled by a unique token in Mutex. Then the site sends messages with the updates to all the other sites, and it releases the token after having received all the acknowledgments.

The two classes of the net are the sites and the grant. The colour set of the places Iddle, Mess, Wait, Update, Ack is the sites class and the colour set of Mutex is the grant class. Initially there is a token of each site colour in Iddle and a token in Mutex.



II ALGEBRAIC STRUCTURE OF R.N. FLOWS

II.1 Some recalls about vector spaces

Definition 1 Let E be a vector space, then B = {v1,…vn} a set of vectors of E is a basis if and only if :

$$\forall v \in E \quad \exists! (a1,\ldots,an) \in \mathbf{Q}^n \quad \text{such that } v = a1.v1 + \ldots + an.vn$$

Definition 2 Let F,G be two subspaces of a vector space, then F+G is the subspace defined by : $F+G = \{ v+w \,/\, v \in F, w \in G \}$
and F+G is a direct sum of F and G if and only if

$$\forall u \in F+G \quad \exists! v \in F \quad \exists! w \in G \text{ such that } u = v + w$$

Proposition Let F,G be two subspaces of a vector space, such that F+G is a direct sum, let B_F be a basis of F and B_G be a basis of G. Then $B_F \cup B_G$ is a basis of F+G.
This may be generalized to a finite number of subspaces.

II.2 Special flows of a regular net

In this paragraph we define different kinds of flows of a regular net and we show that the subspace of each kind of flows is isomorphic to the space of flows of an ordinary Petri net deduced from the initial regular net.

Definition 1 Let R be a regular net and E its vector space of coloured places, then H is the homogeneous subspace of E a basis of which is (p_n), $p \in P$, with $p_n = \Sigma\,(p,c)$, where the summation is over $c \in C(p)$

Proposition 1 Let R be a regular net , let p be a place of R, t be a transition of R and ct be a colour of C(t), let $I(p,t) = a.Si + b.Xi$. Then :

$$I(p,t,ct).p_n = a.ni + b$$

Proof :
Let ct = (c1,…,cn)

$$I(p,t,ct).p_n = \sum_{c \in Ci} I(p,t,(c1,\ldots,cn)).(p,c)$$

$$= \sum_{c \in Ci, c \neq ci} I(p,t,(c1,\ldots,cn)).(p,c) + I(p,t,(c1,\ldots,cn)).(p,ci)$$

$$= (ni-1).a + (a+b) = a.ni + b \quad ‰$$

Definition 2 The underlying Petri net R_n associated to a regular coloured net $R = \langle P,T,C,K,I^+,I^- \rangle$ (obtained by forgetting the colour information) is defined by :

- the set of places P
- the set of transitions T
- Let $I^+(p,t) = a.S_i + b.X_i$ then
  $I_n^+(p,t) = a.ni + b$ where $ni = |Ci|$
- I_n^- is similarly defined

Remark The definition of the underlying net in [Jen82] is based on the multiplicity of a colour function f(p,t) which is denoted by | f | and which counts the number of tokens in a place p if this number is independant from the selected colour of C(t). One can see in our definition that (for example) : $I_n^+(p,t) = a.ni + b = |\, I^+(p,t)\,| = |\, a.S_i + b.X_i\,|$
Hence the two definitions are equivalent.

Proposition 2 Let λ_n be the linear application from E_n the vector space of places of R_n to H defined by : $\lambda_n(p) = p_n$. Then λ_n is an isomorphism between the flows of E_n and the flows of H.

Proof

This proposition is a consequence of the proposition 1. Indeed in order to obtain the flows from the incidence matrix of (p_n) , it is sufficient to keep only one column per transition since all the columns of a transition are equal. Then it is easy to verify that this reduced matrix is exactly the incidence matrix of the underlying net. ‰

Since a flow of H does not keep the colour information it is necessary to also study the different behaviour between the colours of a same class when such a class has more than one element. Let J = { i/ ni > 1 }, for each class Ci with $i \in J$, we choose any reference colour $ci_0 \in Ci$ in order to examin such differences.

Definition 3 Let R be a regular net and E its vector space of coloured places, let $i \in J$ and let $c \in Ci$ be a colour distinct from ci_0 , then D_c is the c-synchronous subspace of E a basis of which is (p_c) for all places p such that C(p) = Ci and where $p_c = (p,c) - (p,ci_0)$.

Proposition 3 Let R be a regular net , let p be a place of R, t be a transition of R and ct = (c1,…,cn) be a colour of C(t), let I(p,t) = a.Si + b.Xi . Then :

$$I(p,t,ct).p_c = b \quad \textbf{if } ci = c$$
$$= -b \quad \textbf{if } ci = ci_0$$
$$= 0 \quad \textbf{otherwise}$$

Proof

$$I(p,t,ct).p_c = I(p,t,(c1,\ldots,cn)).(p,c) - I(p,t,(c1,\ldots,cn)).(p,ci_0)$$
$$= (a+b) - a = b \quad \textbf{if } ci = c$$
$$= a - (a+b) = -b \quad \textbf{if } ci = ci_0$$
$$= a - a = 0 \quad \textbf{otherwise}$$ ‰

Definition 4 The i-synchronous Petri net Ri associated to a regular coloured net R = <P,T,C,I^+,I^-> is defined by :

- the set of places $P_i = \{ p \in P / C(p) = Ci \}$
- the set of transitions T
- Let $I^+(p,t) = a^+. Si + b^+. Xi$, $I^-(p,t) = a^-. Si + b^-. Xi$
  if $b^+ - b^- > 0$ then $I_i^+(p,t) = b^+ - b^-$, $I_i^-(p,t) = 0$
  else $I_i^+(p,t) = 0$, $I_i^-(p,t) = b^- - b^+$

Proposition 4 Let λ_c be the linear application from E_i the vector space of places of R_i to D_c defined by : $\lambda_c(p) = p_c$. Then λ_c is an isomorphism between the flows of E_i and the flows of D_c.

Proof

This proposition is a consequence of the proposition 3. Indeed, in order to obtain the flows from the incidence matrix of (p_c) , it is sufficient to keep only one column per transition since the nul columns and those either equal or opposite to the column kept can be eliminated. Then it is easy to verify that this reduced matrix is exactly the incidence matrix of the i-synchronous net. ‰

Definition 5 For $i \in J$, the i-synchronous subspace $D_i = \sum D_c$ where the summation is over $c \in Ci$, is generated by all the differences between two arbitrary colours of Ci.

Definition 6 The synchronous subspace $D = \sum D_i$ where the summation is over $i \in J$, is generated by all the differences between two arbitrary colours of a same class.

II. 3 Structure of the flows subspace

The following theorems show a detailled decomposition of each flow of E' as sum of an homogeneous flow of H' and synchronous flows of D'_c . Moreover this decomposition is unique. Since proofs are technical and long, we prefer to give sketches of proof. The proofs will be found in [Had87].

Theorem 1 $E' = H' + D'$ and this sum is direct

Sketch of proof Given any flow v, we build a vector w belonging to H by taking for coefficients of this vector the averages on each colour domain of coefficients of v. We show that w is a also a flow, because of the regularity of colour functions. Then by construction, v-w belongs to D and since v and w are flows, v-w is also a flow ‰

Theorem 2 $D' = \sum D'_i$ where the summation is over $i \in J$ and this sum is direct

Sketch of proof Given any flow v of D, we build one vector vi per D_i by projection of v on places p such that C(p) = Ci. We have $v = \sum vi$. The proof that vi is a flow is based on the following point : the tokens evolution of vi by a transition does not depend of the colours distinguished in domains different from Ci ‰

Theorem 3 Let $i \in J$, $D'_i = \sum D'_c$ where the summation is over $c \in Ci$, $c \neq ci_0$ and this sum is direct

Sketch of proof We use a similar construction to the proof of the second theorem and then we study the different cases of selected colour of Ci in a firing of a transition ‰

III ALGORITHM FOR A P.R.N. FLOWS BASIS

Using now the propositions given in II.2 and the theorems given in II.3 , we can obtain a basis of flows of a regular net by computing only :

- A basis of H' i.e. a flows basis of the underlying net.

- $\forall i \in J$, a basis of D'_c for only one $c \in Ci$, i.e. a flows basis of the i-synchronous net. (Indeed the flows of D_c are isomorphic to the flows of the i-sychronous net for any $c \in Ci$ from the proposition II.2.4)

To generalize this method to parametrized nets we have to distinguish the cases where the class cardinalities are fixed or variable.

Notations

- **Z**[N1,…,Nk] denotes the ring of polynoms with k variables and coefficients in **Z**
- **Z***[N1,…,Nk] denotes the subset of non nul polynoms
- V[N1,…,Nk] denotes a matrix or a vector or a family of vectors with coefficients in **Z**[N1,…,Nk]
- V[n1,…,nk] denotes the corresponding matrix (vector, family of vectors) with coefficients in **Z** , ni substituted for Ni

III.1 Homogeneous flows of a P.R.N.

When the class cardinalities are variable the coefficients of the incidence matrix of the underlying net have the form a.Ni+b instead of a.ni+b. Thus these coefficients belong to the ring **Z**[N1,…,Nk] and we can denote this matrix I[N1,…,Nk]. **Z**[N1,…,Nk] being entire and commutative Gauss elimination may be applied to find a basis of flows. Indeed any entire and commutative ring can be imbedded in a field which is called its field of fractions. Moreover Gauss elimination may be applied in this field in order to obtain that the coefficients of the basis belong to the ring. A practical method with evaluation is given in [Kuj84].

Yet, we must solve the following problem : let B[N1,…,Nk] be a flows basis of I[N1,…,Nk] what condition must verify (n1,…,nk) so that B[n1,…,nk] is a flow basis of I[n1,…,nk] ? The following propositions give us a sufficient and computable condition.

Proposition 1 If V[N1,…,Nk] is a flow of I[N1,…,Nk] then V[n1,…,nk] is a flow of I[n1,…,nk]

Proof $\forall$ t $\in$ T V[N1,…,Nk] . I[N1,…,Nk] (t) is the nul polynom.
So V[n1,…,nk] . I[n1,…,nk] (t) is nul ‰

Proposition 2 Let B[N1,…,Nk] be a basis of flows of I[N1,…,Nk] computed by Gauss elimination, then there exists a computable polynom P $\in$ **Z***[N1,…,Nk] such that :

P(n1,…,nk) $\neq$ 0 $\Rightarrow$ B[n1,…,nk] is a flows basis of I[n1,…,nk]

Remark A computable polynom is a polynom the coefficients of which can be computed by an algorithm.

Proof In Gauss elimination each step builds a basis of flows for I reduced to its (q+1) first columns from a basis of flows for I reduced to its q first columns. Our proof follows this recurrence schema :
- Initially $B_0 = (p_n)$ and $P_0 = \mathbf{1}$ the constant polynom
- After the q^{th} step ,

. Let B_q[N1,…,Nk] = {V1,…,Vr} be the basis of flows of I[N1,…,Nk] reduced to q columns

. Let P_q be the polynom found at the q^{th} step, i.e.

$P_q(n1,\ldots,nk) \neq 0 \Rightarrow B_q[n1,\ldots,nk]$ is a flows basis of I[n1,…,nk] reduced to q columns

- For the next step of the algorithm

. t is the $(q+1)^{th}$ transition

. $f(Vi) = Vi.\mathbf{I}[N1,\ldots,Nk](t) \in \mathbf{Z}[N1,\ldots,Nk]$

1) If f(Vi) = 0 for all Vi then the basis and the polynom remains the same :

. $B_{q+1}[N1,\ldots,Nk] = B_q[N1,\ldots,Nk]$, $P_{q+1} = P_q$

2) If $f(Vi) \neq 0$ for at least one Vi, we may suppose $f(V1) \neq 0$:

. A new base of r-1 vectors $B^{q+1}[N1,\ldots,Nk] = \{W2,\ldots,Wr\}$ is computed by Gauss elimination : $Wi = f(V1).Vi - f(Vi).V1$, for i = 2…r

. Let M be the matrix Wi/Vi :

$$\begin{pmatrix} -f(V2) & \cdots & -f(Vr) \\ f(V1) & & 0 \\ & \ddots & \\ 0 & & f(V1) \end{pmatrix}$$

. The sub-determinant SD obtained by deleting the first row verifies : $SD = f(V1)^{r-1}$

. We take $P_{q+1} = P_q \,.\, f(V1)$

. Let (n1,…,nk) be such that $P_{q+1}(n1,\ldots,nk) \neq 0$ then

(i) $P_q(n1,\ldots,nk) \neq 0$ (ii) $f(V1)(n1,\ldots,nk) \neq 0$

(i) $\Rightarrow B_q[n1,\ldots,nk]$ is a flows basis of I[n1,…,nk] reduced to q columns

(ii) $\Rightarrow B_{q+1}[n1,\ldots,nk]$ is a free family since M has a non nul sub-determinant SD

(ii) $\Rightarrow$ The vectorial subspace of flows of **I**[n1,…,nk] reduced to q+1 columns, has dimension r-1 since $f(V1).(n1,\ldots,nk) \neq 0$

. Hence $B_{q+1}[n1,\ldots,nk]$ is a flows basis at the $(q+1)^{th}$ step and P^{q+1} is the required polynom ‰

In fact, we have a more general result given without proof.

<u>Proposition 3</u> Let B[N1,…,Nk] be any basis of flows of **I**[N1,…,Nk] , then there exists a computable polynom $P \in \mathbf{Z}^*[N1,\ldots,Nk]$ such that :

$P(n1,\ldots,nk) \neq 0 \Rightarrow B[n1,\ldots,nk]$ is a flows basis of I[n1,…,nk]

III.2 <u>Synchronous flows of a P.R.N.</u>

As we noticed in the beginning of the paragraph, the computing of a basis of D'_c is made one time per class of colour. Moreover the ni coefficient does not appear in the i-synchronous net. Hence there is no difference between computing synchronous flows in R.N. and in P.R.N.

III.3 <u>Example</u>

We construct now the basis of flows for the net modelling the data base management.At first we give the incidence matrix of this net :

		T1	T2	T3	T4
Wait	:	X1	-X1	0	0
Update	:	0	0	X1	-X1
Iddle	:	-X1	X1	-X1	X1
Mess	:	S1-X1	0	-X1	0
Ack	:	0	-S1+X1	0	X1
Mutex	:	-S2	S2	0	0

1. Now we build the incidence matrix of the underlying net by substituing (a.Ni + b) for (a.Si + b.Xi) :

		T1	T2	T3	T4
W	:	1	-1	0	0
U	:	0	0	1	-1
I	:	-1	1	-1	1
Me	:	N1-1	0	-1	0
A	:	0	-N1+1	0	1
Mu	:	-N2	N2	0	0

Then we apply Gauss elimination step by step on this polynomial matrix, giving the successive values of the pivot and the polynom P :

T1 : { pivot = 1 , P = 1 }

		T2	T3	T4
U	:	0	1	-1
I+W	:	0	-1	1
Me-(N1-1).W	:	N1-1	-1	0
A	:	-N1+1	0	1
Mu+N2.W	:	0	0	0

T2 : { pivot = N1-1 , P = N1-1}

		T3	T4
U	:	1	-1
I+W	:	-1	1
Me+A-(N1-1).W	:	-1	1
Mu+N2.W	:	0	0

T3 : { pivot = 1 , P = N1-1}

		T4
I+W+U	:	0
Me+A+U-(N1-1).W	:	0
Mu+N2.W	:	0

Finally the basis of H' is :

$Iddle_n + Wait_n + Update_n$

$Mess_n + Ack_n + Update_n - (N1-1).Wait_n$

$Mutex_n + N2.Wait_n$

The polynom found for the basis of H' is **N1-1**. Then the basis found is available for any value (n1,n2) such that $n1 \neq 1$. In this particular case, one can see that this basis is available even for n1 = 1 (the polynomial condition is sufficient but not necessary).

2. Now we build the incidence matrix of the synchronised net for the sites by substituing (b) for (a.Si + b.Xi) and only keeping the places the colour set of which is the sites class :

		T1	T2	T3	T4
Wait	:	1	-1	0	0
Update	:	0	0	1	-1
Iddle	:	-1	1	-1	1
Mess	:	-1	0	-1	0
Ack	:	0	1	0	1

Then we apply the classical Gauss elimination (on integer coefficients) and we find a basis of D'_c (c colour of a site) :

$Iddle_c + Wait_c + Update_c$

$Mess_c + Wait_c + Update_c + Ack_c$

CONCLUSION

We have introduced a new subclass of coloured nets, the regular coloured nets and parametrized these nets. Our main result is a fundamental decomposition of the flows space over outstanding subspaces. This result leads to an algorithm for computing a flows basis on a parametrized regular net. Thus we have extended the calculus principles of flows of coloured nets by pointing out the fact that every efficient algorithm to compute flows in high-level nets must be based on the algebraic structure of the flows space.

There are two possible developments for this work. On the one hand the definition of regular nets may be extended with similar results (product of classes, colour successor,…). On the other hand different tools of proof for Petri nets may be also generalized such as the reduction theory [Ber83] or the accessibility graph. We have already developed a symbolic accessibility graph construction [Had86] improving, in the case of regular nets the results given by [Hub85].

ACKNOWLEDGEMENTS

A number of useful suggestions and improvements for this work were made by the three referee reports we have received on this paper.

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