A REDUCTION THEORY FOR COLOURED NETS

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Abstract

This paper presents the generalization for the coloured nets of the most efficient reductions defined by Berthelot for Petri nets. First a methodology of the generalization is given which is independent from the reduction one wants to generalize. Then based on this methodology, we define extensions of the implicit place transformation and the pre and post agglomeration of transitions. For each reduction we prove that the reduced net has exactly the same properties as the original net. Finally we completely reduce an improved modelling of the data base management with multiple copies showing, by this way, its correctness.

INTRODUCTION

In any theoretical model of computation, a useful method to prove properties of an object (program, protocol, …) of this model is to reduce this object such that the simplified object has the same properties as the original one. In his thesis, Berthelot [Ber83] has defined ten reductions of Petri nets and has shown how they can be efficiently used in various modellings by Petri nets (See also [Ber85]).

So as abbreviations of Petri nets - coloured nets [Jen82] and predicate transition nets [Gen81], [Lau85] - are now defined in order to model complex systems, an interesting contribution to high level nets theory would be the definition of similar reduction rules. The main works which have been done are [Col86] , [Kro89] and [Gen88]. In [Col86] the authors define different rules where the main "coloured" condition is the orthonormality of the coloured functions (too strong a condition in our opinion) which valuate some arcs and where the structural conditions are similar but not equivalent to the conditions of Berthelot. In [Kro89] the authors study the extension of reduction rules to Predicate-Transition nets but they only ask for their reductions to preserve local properties. The work of Genrich is a deep study of equivalence for Predicate transitions nets, namely it defines a set of transformation rules (sound and complete) which ensure that the original and the transformed Predicate transition nets have the same unfolded net in all valid interpretations. These rules are complementary to the rules we will define here, since they do not reduce the unfolded net but can extend the application field of our rules by adequate transformations. In fact we will give here a transformation (the orthonromalized reduction) which can be deduced from the Genrich rules and which has allowed us to extend the reductions presented in [Had88].

The work which we present here, leads to alternative definitions for reduction rules which can be added to those defined in [Col86] and provides a more powerful tool. But rather than these additional rules, the main contribution is the principle of the generalization of the reductions. Indeed the reduction theory is nothing but than **an heuristic method** and then :

- It provides **only sufficient conditions** to the simplification problem,
- It will always be improvable **by more sophisticated rules**.

So, in our opinion, the presentation of a methodology to generalize reductions for coloured nets is at least as important as the definition of new reduction rules. The methodology we present here is based on two principles :

- Do not define, if possible, additional structural conditions for the extended reduction rules.
- Only define the functional conditions necessary to ensure the equivalence between the reduced net and the original net.

Preserving these two principles while one generalizes a reduction makes the extended reduction as accurate as the original one. In order to respect these two principles, it is essential while defining and proving the extension to take into account the unfolded Petri net of the coloured net.

In order to illustrate this methodology we have choosen to extend the most frequently used rules of Berthelot, namely the implicit place simplification and the pre and post agglomeration of transitions. We emphasize two advantages of our reductions : on the one side, they are strictly equivalent to the reductions defined by Berthelot and they then have the numerous properties proved by him; on the other side the functional conditions are not predefined but are the weakest possible necessary to obtain this equivalence in each case and thus giving them a large field of application.

The coloured reductions we have defined, completely reduce an improved version [Had87b] of the data base management [Jen82] with multiple copies. In [Had87b], one can also find the complete reduction of the two-step comitment protocol [Bae81]. These reductions have been also shown to be programmable for the regular coloured nets in this thesis.

General notations

- **N** is the set of non negative integers
- **Z** is the set of integers
- **Q** is the set of rational numbers
- M.N , where M and N are matrices, denotes the matrices product (this notation includes the product of a vector by a matrix, since a vector is a special case of matrix)
- M^t , where M is a matrix n x p, denotes the matrix p x n such that : $M^t_{i,j} = M_{j,i}$
- Let U be a finite set. Then the set of functions from U to **N** is denoted Bag(U). An item a of Bag(U) is noted $\sum a_u.u$ where the summation is over $u \in U$.
- A partial order on Bag(U) is defined by : $a \geq b$ if and only if $\forall u \in U$, $a_u \geq b_u$
- The sum of two items of Bag(U) is defined by $a+b = \sum (a_u+b_u).u$ where the summation is over $u \in U$
- The difference between two items $a \geq b$ of Bag(U) is defined by : $a-b = \sum (a_u-b_u).u$ where the summation is over $u \in U$

0 COLOURED NETS

We recall the definitions of a coloured net, the firing rule in a coloured net, some particular coloured functions, the equivalent unfolded Petri net and the flows definition that we need for the implicit places rule.

Definition 1 A coloured net $R = <P,T,C,I^+,I^-,M>$ is defined by :

- P the set of places
- T the set of transitions with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$
- C the "colour function" from P U T to Ω , where Ω is some finite set of finite and not empty sets. An item of C(s) is called a colour of s and C(s) is called the colour set of s.
- I^+ (I^-) is the forward (backward) incidence matrix of P x T , where $I^+(p,t)$ is a function from C(p) x C(t) to **N** (i.e. a linear application from Bag(C(t)) to Bag(C(p)))
- M the "initial marking" of the net is a vector of P, where M(p) is a function from C(p) to **N** (i.e. an item of Bag(C(p)))

Notation

We note $I^+,I^-(p,t)(c_t)$, where c_t belongs to C(t), the corresponding item of Bag(C(p)).

Definition 2 The firing rule is defined by :

- A transition t is enabled for a marking M and a colour $c_t \in C(t)$ if and only if :

  $\forall p \in P$, $M(p) \geq I^-(p,t)(c_t)$
- The firing of t for a marking M and a colour $c_t \in C(t)$ gives a new marking M' defined by : $\forall p \in P$, $M'(p) = M(p) - I^-(p,t)(c_t) + I^+(p,t)(c_t)$

We present the particular functions we need for the definition of coloured reductions. As we have already said, a colour function can be defined either as a linear application from Bag(C(t)) to Bag(C(p) or as a function from C(p) x C(t) to **N** . As we need the two definitions for this paper we use the same symbol for the two functions and the formula below show how to translate one definition to the other :

$f(c) = \sum f(c',c).c'$ where c' ranges over C(p)

where f(c) denotes the mapping of c to an item of Bag(C(p) by f as a linear application and where f(c',c) denotes the mapping of (c',c) to an integer value. Notice that no confusion can appear since the first definition implies one argument while the second definition implies two arguments. In the next definitions, all the functions are linear applications.

Definition 3 The identity function of Bag(C) "Id" is defined by Id(c) = c

Remark : The second definition gives Id(c',c) == If c=c' then 1 else 0

Definition 4 A function f from Bag(C) to Bag(C) is orthonormal if and only if there exists a substitution σ of C such that $f(c) = \sigma(c)$

Remark : The second definition gives f(c',c) == If $\sigma(c)=c'$ then 1 else 0

Definition 5 The projection from Bag(CxC') to Bag(C) "Proj" is defined by :
$Proj(<c,d>) = c$
Remark :

- This definition is not the usual definition of projection on vector spaces
- The second definition gives $Proj(c',<c,d>)$ == If c=c' then 1 else 0

Definition 6 A function f from Bag(C) to Bag(C') is quasi injective if and only if :

(Let us recall that $f(c) = \sum f(c',c).c'$ where c' ranges over C')

$\forall c' \in C', \forall c_1 \in C, \forall c_2 \in C$, $f(c', c_1) \neq 0$ and $f(c, c_2) \neq 0 \Rightarrow c_1 = c_2$

Definition 7 A function f from Bag(C) to Bag(C') is unitary if and only if :

(Let us recall that $f(c) = \sum f(c',c).c'$ where c' ranges over C')

$\forall c' \in C', \forall c \in C$ $f(c',c) = 0$ or $f(c',c) = 1$

In a modelling the functions are almost always unitary while projections, orthonormal functions and identities are currently used. The quasi injectivity is a significant property for the unfolding of the coloured net and is fulfilled by a wide class of standard coloured functions.

Definition 8 Let f be a function from Bag(C) to Bag(C') and g be a function from Bag(C') to Bag(C"). Then the composition of f and g is a function g o f from Bag(C) to Bag(C") defined by :

$g \circ f(c) = g(f(c)) = \sum (\sum g(c'',c').f(c',c)) . c''$

where c' ranges over C' and c" ranges over C".

We now define the unfolded Petri net of a coloured net as in [Jen81b]. This net is the low level representation of the coloured net, has exactly the same behaviour as the coloured net and then properties of the ordinary net are properties of the coloured net. This equivalence is fundamental since it is the basis of the theory of high level nets.

Definition 9 Let R be a coloured net then R' the unfolded Petri net of R is defined by :

- P' = U (p,c) where the union is over $p \in P$ and $c \in C(p)$
- T' = U (t,c) where the union is over $t \in T$ and $c \in C(t)$
- I'^- and I'^+ the backward and the forward matrices are defined by :
  $I'^-(p,c)(t,c') = I^-(p,t)(c,c')$ and $I'^+(p,c)(t,c') = I^+(p,t)(c,c')$
- M'(p,c) = M(p)(c)

We need for the implicit place transformation, the definition of the flows of the coloured net. There are different ways to define them. Here we choose a simple definition : the flows of the coloured net are the flows of the unfolded Petri net.

Definition 10 The incidence matrix I of a Petri net is defined by : $I = I^+ - I^-$. Then I(p,t) belongs to **Z**

Definition 11 A flow v of a Petri net is a vector of $\mathbf{Z}^p$ (where P is the set of places) which verifies : $I^t.v = 0$. The support of a flow is the subset P' of P defined by :

$$p \in P' \iff v_p \neq 0$$

Definition 12 A flow v of a coloured net R is a flow of the unfolded Petri net of R.

Remark Sometimes depending on the authors the flows are called invariants. We prefer our notation since invariants are not necessary linear and even if they are linear they do not necessary verify the flow equation (See definition 11).

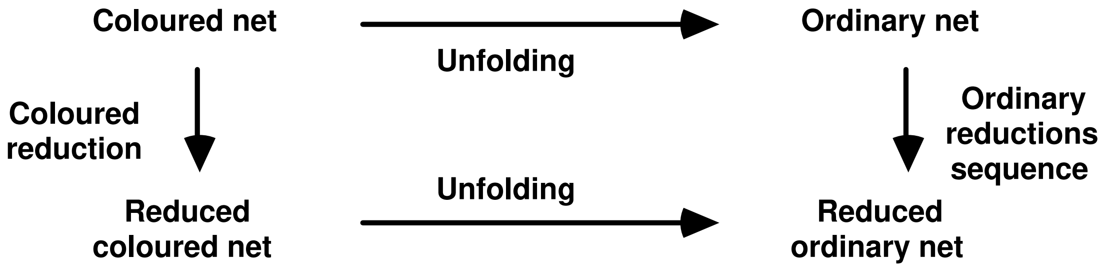
At last , all the reductions we will define here preserve the main properties of a net. So we collect them in a set.

Definition 13 The set of main properties of Petri net is defined by :
{boundness, safeness, invariant covering, normal end, home state, unavoidable state, liveness, pseudo-liveness, quasi-liveness, abstraction properties}

Remark For more informations about these properties the reader may refer to [Bra83] and especially for the abstraction properties to [And81]

1 PRINCIPLE OF THE EXTENSION OF A REDUCTION

The schema below summarizes the process we have followed to specify and to validate an coloured extension of an ordinary reduction.



a. Specifying a coloured reduction

As for the corresponding ordinary reduction, the coloured reduction is defined by two specifications, the application conditions and the transformation rule.

When specifying a coloured reduction , the application conditions have to be decomposed in two ways :

- Structural reductions which (if possible) must be the same ones as for the ordinary reduction. Example : a transition does not share its input places. (Cf the ordinary pre-agglomeration)
- Functional conditions which can not be predefined but are just those necessary to ensure a "good" unfolding. Example : a quasi-injective function valuating an arc from a place to a transition ensures that each unfolded transition does not share its input places. (Cf the coloured pre-agglomeration)

The transformation rule must verify these two principles :

- It may not "increase" the coloured domain of the transitions of the net. (this is an imperative condition in order to have practical use of a coloured reduction)
- It only allows the composition and inverse of coloured functions in order to build new valuations since these are the only significant operations over coloured functions.

For instance we disallow the use of generalized inverses since a function may have multiple generalized inverses and moreover their signification (for the behaviour of the net) is unclear.

b. Validating the coloured reduction

Once the reduction is defined, it remains to prove that the preceeding diagram comutes. That is to say , each of the three steps (unfolding of the original net, reductions sequence, unfolding of the reduced net) is implicitly done in the specification of the coloured reduction. Let us detail it :

Unfolding of the original coloured net

One must recognize in the unfolded net the application conditions of one or more ordinary reductions as consequences of the application conditions in the coloured net.

Reductions sequence

One must verify that once an ordinary reduction is applied in the unfolded net, the conditions of reductions not yet applied are still true.

Unfolding of the reduced coloured net

Once all the ordinary reductions have been done, one must prove that the reduced net is the unfolded net of the coloured reduced net.

2 IMPLICIT PLACE SIMPLIFICATION

2.1 Ordinary implicit place simplification [Ber83]

From the original definition of an implicit place, we have excluded the case of multiple initial markings and unbounded implicit places which can be found by the covering graph and which generally represents a serious fault in the modelling. As this definition is a restricted definition, all the results remain true. We have also suppressed a condition from the Berthelot definition, since this condition is never used in the proofs and thus, in our opinion, is unnecessary.

Definition 1 **Implicit place**

Let (R,Mo) be a marked Petri net, a place p of R is implicit related to a subset of places P' if and only if :

(1) There is a flow f the support of which is {p} U P' :

$$f = a_p.p - \sum_{q \in P'} a_q.q \text{ with } a_p,a_q \in \mathbf{N}$$

(2) $$\forall t \in T,\ a_p.I^-(p,t) - \sum_{q \in P'} a_q.I^-(q,t) \leq a_p.Mo(p) - \sum_{q \in P'} a_q.Mo(q)$$

Interpretation An implicit place will never disable the firing of a transition since it initially does not disable it because of (2) and this condition is reproducible for all the reachable markings because of (1).

Definition 2 **Implicit place simplification**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by simplification of the implicit place p is defined by :

- $P_r = P - \{p\}$
- $T_r = T$
- $\forall t \in T_r$, $\forall p' \in P_r$, $I_r^-(p',t) = I^-(p',t)$ et $I_r^+(p',t) = I^+(p',t)$
- $\forall p' \in P_r$, $Mo_r(p') = Mo(p')$

Interpretation One deletes the implicit place (arcs and marking included)

Theorem Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by simplification of the implicit place, π a main property different from safeness. Then :

(R,Mo) verifies π <=> (R_r,Mo_r) verifies π

(R,Mo) is safe => (R_r,Mo_r) is safe

Proof in [Ber83]

2.2 Coloured implicit place simplification

In contrast to the other reductions that we will present here, the implicit place is based on a algebraic property (existence of a particular flow). Then the generalization of this reduction implies the existence of flows computation for coloured nets (See for instance [Had86] , [Had87b] or [Sil85]). Since the computation of flows for coloured nets is much more complex than in ordinary Petri nets, this reduction, which could by hand be done in Petri nets, now needs the help of a good flows computation. We have not choosen a behavioural definition of an implicit place (as it is done in [Col86]) since the verification of such conditions needs the examination of all reachable markings !

Definition 1 **Coloured implicit place**

Let (R,Mo) be a coloured net, a place p is implicit if and only if :

(1) $\forall c \in C(p)$, There is a flow f_c the support of which is $\{(p,c)\} \cup P'$ where $P' = \{ (q_1,c_1),\ldots,(q_k,c_k) \}$

$$f_c = a_{pc}.(p,c) - \sum_{i=1\ldots k} a_i.(q_i,c_i) \text{ with } a_{pc},a_i \in \mathbf{N} \text{ and } \forall c' \in C(p), (p,c') \in P'$$

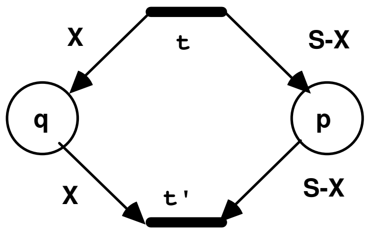
(2) $\forall t \in T, \forall c_t \in C(t)$

$$a_{pc}.I^-(p,t)(c,c_t) - \sum_{i=1\ldots k} a_i.I^-(q_i,t)(c_i,c_t) \le a_{pc}.Mo(p,c) - \sum_{i=1\ldots k} a_i.Mo(q_i,c_i)$$

Example

$C(t) = C(t') = C(p) = C(q) = \{c_1,\ldots,c_n\}$, $Mo(p) = Mo(q) = 0$

$X(c_i) = c_i$ and $(S\text{-}X)(c_i) = \sum_{i \neq j} c_j$



Then $f_i = (p,c_i) - \sum_{i \neq j} (q,c_j)$ and p is implicit.

Notice that even on two places the computing of this flow is not obvious and that an inattentive reader may believe q is the implicit place ! Let us prove that q is not implicit (n=3) :

Mo[t(c1).t(c2)>M1 with M1(q)=c1+c2 and M1(p)=c1+c2+2.c3.

Then t'(c3) is not enabled for M1 because of M1(q)(c3)= 0 while $M1(p) \geq c1+c2$

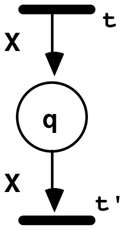
As in ordinary Petri nets, the transformation deletes the implicit place and its arcs.

Definition 2 **Implicit coloured place simplification**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by simplification of the implicit place p is defined by :

- $P_r = P - \{p\}$
- $T_r = T$
- $\forall t \in T_r , \forall p' \in P_r , C_r(p') = C(p')$ and $C_r(t) = C(t)$
- $\forall t \in T_r , \forall p' \in P_r , I_r^-(p',t) = I^-(p',t)$ and $I_r^+(p',t) = I^+(p',t)$
- $\forall p' \in P_r , Mo_r(p') = Mo(p')$

Example (continued)



Theorem Let (R,Mo) be a coloured net and (R_r,Mo_r) be the reduced net by simplification of implicit place, then the unfolded net of (R_r,Mo_r) is obtained by a sequence of simplification of implicit places starting from the unfolded net of (R,Mo).

Proof

Step 1

Let (R',Mo') be the unfolded net of (R,Mo). In this net each place (p,c) has the corresponding invariant :

$$a_{pc}.(p,c) - \sum_{i=1\ldots k} a_i.(q_i,c_i) \text{ with } a_{pc},a_i \in \mathbf{N} \text{ and } \forall c', (p,c') \in P'$$

with

$$\forall t \in T, \forall c_t \in C(t)$$

$$a_p.I'^{-}[(p,c)(t,c_t)] - \sum_{i=1\ldots k} a_i.I'^{-}((q_i,c_i)(c_i,c_t)) \le a_p.Mo'(p,c) - \sum_{i=1\ldots k} a_i.Mo'(q_i,c_i)$$

Hence every place (p,c) fulfills the conditions of an implicit place.

Step 2

Moreover the suppression of a place (p,c) does not change the conditions of the other places, since (p,c) **does not belong** to the support of any flow $f_{c'}$. So one can successively apply the suppression of implicit place to every (p,c).

Step 3

Then the reduced net is exactly the unfolded net of (R_r,Mo_r) since the reductions have not changed the initial marking and the incidences of the other places. ‰

Corollary Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by coloured simplification of the implicit place, π a main property different from safeness. Then :

(R,Mo) verifies π <=> (R_r,Mo_r) verifies π

(R,Mo) is safe => (R_r,Mo_r) is safe

3 ORTHONORMALIZATION

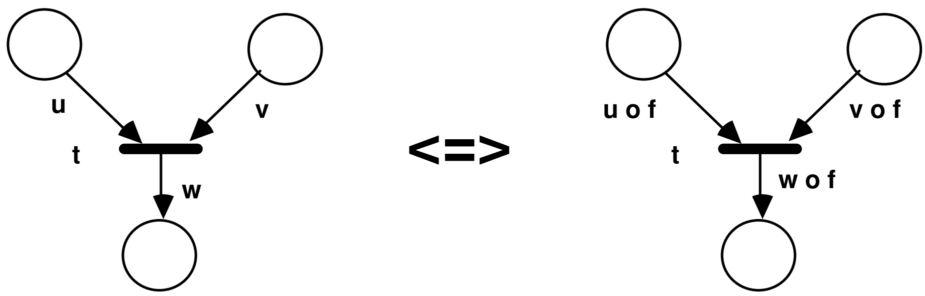
The reduction , we present now, is not a generalization of an ordinary reduction. However it is a very useful one, since it allows to extend the application conditions of the other reductions. In fact this reduction can be wieved as an equivalent transformation [Gen88] and can be proven by the rules presented in this paper. For sake of simplicity, we have choosen to directly prove its correctness. The principle of this reduction is the following : a renaming of the colours of a transition induced by an orthonormal function of the colour domain of this transition.

Definition **Orthonormalization of a transition**

Let (R,Mo) be a marked coloured net, t be a transition of R and f be an orthonormal function of C(t) then the reduced net (R_r,Mo_r) obtained from the net (R,Mo) by the f-orthonormalization of t is defined by :

- Pr = P
- Tr = T
- $\forall$ t $\in$ Tr, $\forall$ p $\in$ Pr, $C_r(t) = C(t)$ and $C_r(p) = C(p)$
- $\forall$ t' $\in$ Tr- {t} , $\forall$ p $\in$ Pr, $I_r^+(p,t') = I^+(p,t')$, $I_r^-(p,t') = I^-(p,t')$
- $\forall$ p $\in$ P_r , $I_r^-(p,t) = I^-(p,t) \circ f$, $I_r^+(p,t) = I^+(p,t) \circ f$
- $\forall$ p' $\in$ P_r, $Mo_r(p') = Mo(p')$

Example



Theorem Let (R,Mo) be a marked coloured net and (R_r,Mo_r) be the reduced net obtained from the net (R,Mo) by the f-orthonormalization of t , then the unfolded nets of these two nets are identical up to an isomorphism which is the identity for the coloured places and the coloured transitions different from (t,c) and which maps (t,c) on (t, σ^{-1} (c)) where σ is the substitution associated to f.

Proof

Let us denote I'^+ (I'^-) the forward (backward) incidence matrix of the unfolded net of (R,Mo) and I'^+_r (I'^+_r) the forward (backward) incidence matrix of the unfolded net of (R_r,Mo_r) . Then we only have to verify that :

$I'^-_r((p,c),(t, \sigma^{-1} (c'))) = I'^-((p,c),(t,c'))$ and $I'^+_r((p,c),(t, \sigma^{-1} (c'))) = I'^+ (p,c),(t,c'))$

$I'^-_r((p,c),(t, \sigma^{-1} (c'))) = I_r^-(p,t)(c, \sigma^{-1} (c')) = I^-(p,t) \circ f (c, \sigma^{-1} (c'))$

$= \bullet\ I^-(p,t) (c,c'').f(c'', \sigma^{-1} (c'))$ where c'' ranges over C(t)

$= I^-(p,t) (c,c') = I'^-((p,c),(t,c'))$

The proof of the second identity is similar ‰

Corollary Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by orthonormalization of a transition, π a main property. Then :

(R,Mo) verifies π <=> (R_r,Mo_r) verifies π

4 PRE-AGGLOMERATION

4.1 Ordinary pre-agglomeration [Ber83]

Definition 1 **Pre-agglomerable transitions**

Let (R,Mo) be a marked Petri net, a subset of transitions F is pre-agglomerable if and only if there is a place p and a transition $h \in F$ such that the following conditions are fulfilled :

(1) $I^+(p,h) = 1$ and $\forall t \neq h$, $I^+(p,t) = 0$
$\forall f \in F$, $I^-(p,f) = 1$ et $\forall t \in F$, $I^-(p,t) = 0$
$Mo(p) = 0$
{ The single input transition of p is h and the output transitions of p are F}
{ All the arcs surrounding p are valuated by 1 }
{ p is unmarked }

(2) $\forall p' \neq p$, $I^+(p',h) = 0$ { The single output place of h is p }

(3) $\exists p' \in P$,such that $I^-(p',h) \neq 0$ { h has an input place }

(4) $\forall p' \in P$, $\forall t \in T- \{h\}$, $I^-(p',h) \neq 0 \Rightarrow I^-(p',t) = 0$
{ h does not share its input places }

Interpretation

p is an intermediate state accessed by the firing of h and left by the firing of any transition of F. The principle of the pre-agglomeration is the following : in every sequence of firings with an occurrence of h followed later by an occurrence of a transition f of F, one can postpone the firing of h and "merge" it with the firing of f.

Definition 2 **Pre-agglomeration of transitions**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by pre-agglomeration of h and F is defined by :

- $P_r = P - \{p\}$
- $T_r = T - \{h\}$
- $\forall t \in T_r / F$, $\forall p' \in P_r$, $I_r^-(p',t) = I^-(p',t)$ et $I_r^+(p',t) = I^+(p',t)$
- $\forall f \in F$, $\forall p' \in P_r$, $I_r^-(p',f) = I^-(p',f) + I^-(p',h)$ et $I_r^+(p',f) = I^+(p',f)$
- $\forall p' \in P_r$, $Mo_r(p') = Mo(p')$

Interpretation The transition h disappears since in the reduced net it is merged with each transition of F. The reduced incidence matrices take this merging into account.

Theorem Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by pre-agglomeration of transitions, π a main property. Then :

(R,Mo) verifies $\pi \Leftrightarrow (R_r,Mo_r)$ verifies π

Proof in [Ber83]

4.2 Coloured pre-agglomeration

In order to define the conditions of a coloured pre-agglomeration, there must be, as in ordinary Petri nets a place p, a transition t and a set of transitions F verifying the structural conditions of the ordinary pre-agglomeration. We are going to explain (before the proof) the additional functional conditions :

- The valuation of the arc between h and p must be an orthonormal function (u) since it implies that in the unfolded net, each place (p,c) has only one input transition with valuation 1 namely $(h,u^{-1}(c))$.

- The valuation of an arc between p and any transition of F must be an unitary function since it implies that in the unfolded net, each arc between (p,c) and (f,c') with $f \in F$ has valuation 1.

- The valuation of an input arc of h must be a quasi injective function since it implies that in the unfolded net, each transition (h,c) does not share its input places.

Definition 1 **Pre-agglomerable transitions**

Let (R,Mo) be a marked coloured net, a subset of transitions F is pre-agglomerable if and only if there is a place p and a transition $h \in F$ such that the following conditions are fulfilled :

(1) $\forall t \neq h$, $I^+(p,t) = 0$ and $\forall t \in F$, $I^-(p,t) = 0$
$C(p) = C(h)$ and $I^+(p,h)$ is an orthonormal function
$\forall f \in F$, $I^-(p,f) \neq 0$ and $I^-(p,f)$ is an unitary function
$Mo(p) = 0$

(2) $\forall p' \neq p$, $I^+(p',h) = 0$

(3) $\exists p' \in P$, such that $I^-(p',h) \neq 0$

(4) $\forall p' \in P$, $\forall t \in T-\{h\}$,
$I^-(p',h) \neq 0 \Rightarrow I^-(p',t) = 0$ and $I^-(p',h)$ is a quasi injective function

Comparison If we compare our reduction rule with the reduction rule n° 8 given in [Col86], we can observe that our rule extends the rule n° 8 :

- In our rule, p may have severals ouput transitions (the subset F) while in the other rule, only a single transition is possible.

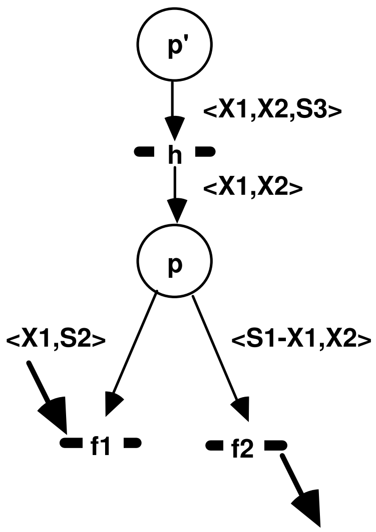
- In our rule, the coloured functions from p to F are unitary while they are orthonormal in the other rule (an orthonormal function is always an unitary function).

However in our rule, there are conditions (quasi-injectivity) on the coloured functions valuating arcs going to h while there are none in the other rule. But then it can be proved with a simple counter-example that without these supplementary conditions, the rule n° 8 does not ensure the equivalence of liveness for the two nets.

<u>Example</u>

$C(p') = C1 \times C2 \times C3$ $C(h) = C(p) = C(f1) = C(f2) = C1 \times C2$
The coloured functions are defined as usual, for instance :
$\langle X_1,X_2,S_3\rangle(c_1,c_2) = \sum_{c \in C3} (c_1,c_2,c)$ and $\langle S_1\text{-}X_1,X_2\rangle (c_1,c_2) = \sum_{c \in C1,\, c \neq c1} (c,c_2)$

The reader may verify that $\langle X_1,X_2,S_3\rangle$ is quasi-injective and $\langle X_1,S_2\rangle$ and $\langle S_1\text{-}X_1,X_2\rangle$ are unitary.



As in the ordinary pre-agglomeration the place p and the transition h disappear. The input arcs of h now become input arcs for each transition f of F but the functions valuating these arcs are successively composed by the inverse of the function valuating the arc between h and p and the function valuating the arc between p and f.

<u>Definition</u> 2 **Pre-agglomeration of transitions**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by a coloured pre-agglomeration of h and F is defined by :

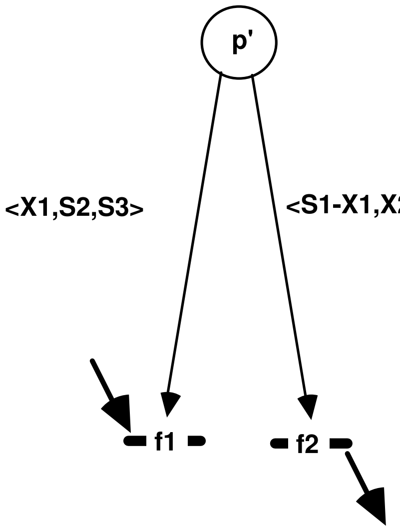
- $Pr = P - \{p\}$
- $Tr = T - \{h\}$
- $\forall t \in Tr, \forall p' \in Pr, C_r(t) = C(t)$ and $C_r(p') = C(p')$
- $\forall t \in Tr, \forall p' \in Pr,\ I_r^+(p',t) = I^+(p',t)$

Let $P_h = \{ p' \in Pr \,/\, I^-(p',h) \neq 0 \}$

- $\forall t \in T, \forall p' \in P_h,\ I_r^-(p',t) = I^-(p',t)$
- $\forall f \in F, \forall p' \in P_h,\ I_r^-(p',f) = I^-(p',h) \circ I^+(p,h)^{-1} \circ I^-(p,f)$
- $\forall p' \in P_r, Mo_r(p') = Mo(p')$

Example (continued)

For this kind of functions the composition can symbolically be done by substitution (See [Had87b]). In our example :
$<S_1-X_1,X_2,S_3> = <X_1,X_2,S_3> \circ <S_1-X_1,X_2>$
$<X_1,S_2,S_3> = <X_1,X_2,S_3> \circ <X_1,S_2>$



Theorem Let (R,Mo) be a coloured net and (R_r,Mo_r) be the reduced net by a pre-agglomeration of transitions, then the unfolded net of (R_r,Mo_r) is obtained by a sequence of pre-agglomerations starting from the unfolded net of (R,Mo).

Proof
The proof is decomposed in two parts. First we prove the theorem in the case where the orthonormal functions are identities. Next we prove that the general case can be reduced to the particular case.
Part A $I^+(p,h)$ is an identity function
Step 1 of part A
Let us verify that the transition (h,c) is pre-agglomerable in the unfolded net :
- Its single output place is (p,c) which only has (h,c) for input transition. This place is unmarked. The valuation of the arc which is between these two nodes is 1. (because of the identity function valuating the arc h -> p and the structural conditions)
- The output transitions of (p,c) are the transitions (f,c') which verify the conditions (a) and (b)
(a) $f \in F$
(b) $I^-(p,f)$ $(c,c') \neq 0$ and then equal to 1 (since $I^-(p,f)$ is unitary)
- The input places of (h,c) are among (p',c') where p' is any input place of h in the coloured net. Since the p' has h for single output transition and since $I^-(p',h)$ is a quasi injective function, if (p',c') is an input place of (h,c) then (h,c) is the single output transition of (p',c').

Step 2 of part A

Let us verify that the reduction applied to (h,c) does not change the conditions of the pre-agglomeration of any (h,c'). It suffices to show that (h,c) and (h,c') do not share a "neighbouring" place :

- (p,c) may not be an input place of (h,c') since p is not an input place of h in the coloured net and (p,c) may not be an output place of (h,c') since its single input transition is (h,c).
- Let (p',c') be an input place of (h,c). Then (p',c') may not be an output place of (h,c') since in the coloured net p' is not an output place of h and (p',c') may not be an input place of (h,c') since (h,c) does not share its input places.

Hence one can successively apply all the pre-agglomerations.

Step 3 of part A

Let us have a look at the reduced net .

- All the transitions (h,c) have disappeared
- All the places (p,c) have disappeared
- Let $q \neq p$ such that q is not an input place of h in the coloured net. Then for each $c \in C(q)$, all the arcs of (q,c) are unchanged.
- Let p' be an input place of h in the coloured net, then for every place (p',c) all its input arcs are unchanged and it has an output arc for each (f,c) such that
  a) $\exists$ c" such that $I^-(p',h)(c,c'') \neq 0$
  (this c" is unique because of the quasi injectivity)
  b) $f \in F$
  c) $I^-(p,f)(c'',c') = 1$

The valuation of this arc is $I^-(p',h)(c,c'')$ and since c" is unique, it can be rewritten as : $\sum_{d \in C(h)} I^-(p',h)(c,d) . I^-(p,f)(d,c') = I^-(p',h) \circ I^-(p,f)(c,c')$

Then this reduced net is clearly the unfolded net of the coloured reduced net.

Part B

First we reduce the coloured net by an orthonormalization of h where the orthonormal function is $I^+(p,h)^{-1}$. Then the reduced net verifies the conditions of the part A and it is easy to see that the final net obtained by the reductions in the part A is the reduced coloured net ‰

<u>Corollary</u> Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by a coloured pre-agglomeration, π a main property. Then :

$$(R,Mo) \text{ verifies } \pi \iff (R_r,Mo_r) \text{ verifies } \pi$$

5 <u>POST-AGGLOMERATION</u>

5.1 <u>Ordinary post-agglomeration</u> [Ber83]

From the original definition of post-agglomeration, we have excluded the case of heterogeneous valuations since they do not appear in practice and they lead to technical complications. As this definition is a restricted definition, all the results remain true.

Definition 1 **Post-agglomerable transitions**

Let (R,Mo) be a marked Petri net, a subset of transitions F is post-agglomérable if and only if there is a place p and a subset of transitions H with $H \cap F = \emptyset$ such that the following conditions are fulfilled :

(1) $\forall\ h \in P$, $I^+(p,h) = 1$ and $\forall\ t \in H$, $I^+(p,t) = 0$

$\forall\ f \in F$, $I^-(p,f) = 1$ and $\forall\ t \in F$, $I^-(p,t) = 0$

$Mo(p) = 0$

{ The input transitions set of p is H and the ouput transitions set of p is F }

{ All the arcs surrounding p are valuated by 1 }

{ p is unmarked }

(2) $\exists\ f \in F$, $\exists\ p' \in P$, such that $I^+(p',f) \neq 0$

{ There is a transition of F which has an output place }

(3) $\forall\ f \in F$, $\forall\ p' \neq p$, $I^-(p',f) = 0$

{ The single input place of every transition of F is p }

Interpretation

p is an intermediate state accessed by the firing of any transition of H and left by the firing of any transition of F. The principle of post-agglomeration is the following : in every sequence of firings with an occurrence of a transition h of H followed later by an occurrence of a transition f of F, one can fire f immediately after the firing of h.

Definition 2 **Post-agglomeration of transitions**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by post-agglomeration of H and F is defined by :

- $P_r = P - \{p\}$
- $T_r = T \cup (H \times F) / (H \cup F)$
- $\forall\ f \in F$, $\forall\ h \in H$, one denotes hf the transition (h,f) de H x F
- $\forall\ t \in T_r - (H \times F)$, $\forall\ p' \in Pr$, $I_r^-(p',t) = I^-(p',t)$ and $I_r^+(p',t) = I^+(p',t)$
- $\forall\ h \in H$, $\forall\ f \in F$,
- $\forall\ p' \in P_r$, $I_r^-(p',hf) = I^-(p',h)$ and $I_r^+(p',hf) = I^+(p',h) + I^+(p',f)$
- $\forall\ p' \in P_r$, $Mo_r(p') = Mo(p')$

Interpretation

The transitions of H and F disappear since they are merged by the cartesian product in the reduced net. The reduced incidence matrices take into account this product.

Theorem Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by post-agglomeration of transitions, π a main property. Then :

$$(R,Mo) \text{ verifies } \pi \Longleftrightarrow (R_r,Mo_r) \text{ verifies } \pi$$

Proof in [Ber83]

5.2 Post-agglomeration with multiple outputs

In order to define the conditions of a coloured post-agglomeration, there must be, as in ordinary Petri nets a place p, a set of transitions H and a set of transitions F verifying the structural conditions of the ordinary post-agglomeration. We are going to explain (before the proof) the additional functional conditions :

- The colour domain of each transition of F must be the same as the colour domain of p and moreover the colour domain of p must be a projection of the colour domain of each transition of H.
- The coloured function valuating each arc from any transition of H to p must be the composition of the projection function and an orthonormal function (u_h) and the coloured function valuating each arc from p to any transition of F must be an orthonormal function (v_f).
- So when a transition (h,<c,c'>) is fired it gives a single token < u_h(c)> in p which can be used only by the firing of some transition (f, $v_f^{-1} \circ u_h(c)$) wher f ranges over F. Then the cartesian product H x F will be well defined in the unfolded net.

Definition 1 **Post-agglomerable transitions - with multiple outputs -**

Let (R,Mo) be a coloured Petri net, a subset of transitions F is post-agglomerable if and only if there is a place p and a subset of transitions H with $H \cap F = \emptyset$ such that the following conditions are fulfilled :

(1) $\forall t \in H$, $I^+(p,t) = 0$ and $\forall t \in F$, $I^-(p,t) = 0$

$\forall h \in H$, $\exists C_h$ such that $C(h) = C(p) \times C_h$

and $I^+(p,h)$ is the composition of u_h , an orthonormal function of C(h), and the projection of C(h) over C(p)

$\forall f \in F$, C(f) = C(p) and $I^-(p,f)$ is an orthonormal function we call it v_f.

Mo(p) = 0

(2) $\forall c \in C(p)$, $\exists f \in F$, $\exists p' \in P$, such that $I^+(p',f)(c) \neq 0$

(3) $\forall f \in F$, $\forall p' \neq p$, $I^-(p',f) = 0$

Remark In the second condition $I^+(p',f)(c)$ is an item of Bag(p') (Cf the notations)

Comparison If we compare our reduction rule with the reduction rule n° 2 given in [Col86], we can observe that our rule extends the rule n° 2 :

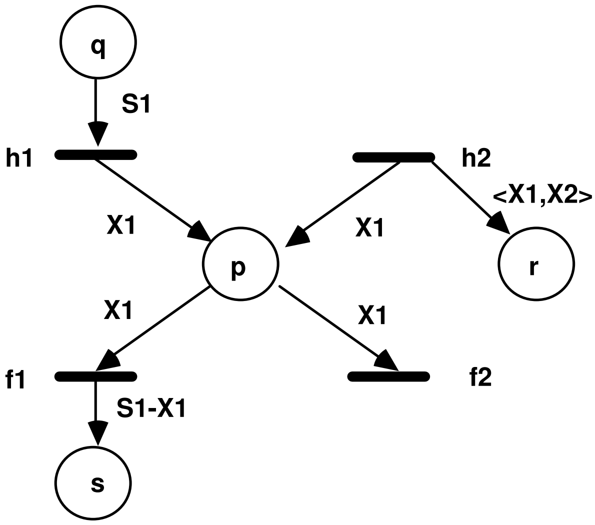
- In our rule, p may have severals ouput transitions (the subset F) and several input transitions (the subset H) while in the other rule, only a single output transition and a single input transition are possible.
- In our rule, the coloured functions from H to p are compositions of an orthonormal function and a projection while in the other rule they are orthonormal. (as the identity is a special case of projection, our category of functions is larger than the one of the rule n°2)

Example

C(h1) = C1 , C(h2) = C1 x C3 , C(f1) = C(f2) = C(p) = C(s) = C1

C(q) = C2 , C(r) = C1 x C3

The coloured functions are defined as usual. Notice that the symbol X1 valuating the arc h2->p denotes a projection since C(h2) = C(p) x C2 while the same symbol X1 valuating the arc p->f1 denotes identity since C(f2) = C(p) (i.e. the denotation of functions symbol in coloured nets depends on the domain of the transitions).



In the post-agglomeration with multiple outputs, the place p disappears and one substitutes the "product" transitions of H x F to the transitions of H and F. The arcs of these transitions are obtained by the union of the arcs of H and F where the valuation of the output arcs of F are successively composed by the inverse of the function valuating the arc between p and f and the function valuating the arc between h and p. The multiple ouputs denote the set F.

<u>Definition</u> 2 **Post-agglomeration of transitions with multiple outputs**

The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by a coloured post-agglomeration of H and F is defined by :

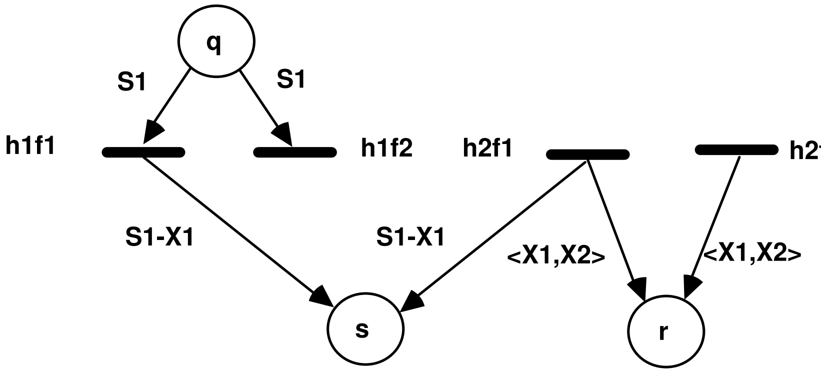
- $P_r = P - \{p\}$
- $T_r = T \cup (H \times F) / (H \cup F)$

  $\forall f \in F, \forall h \in H$, one notes hf the transition (h,f) of H x F
- $\forall t \in T_r / (H \times F), \forall p' \in Pr, C_r(t) = C(t)$ and $C_r(p') = C(p')$

  $\forall f \in F, \forall h \in H, C_r(hf) = C(h)$
- $\forall t \in T_r / (H \times F), \forall p' \in P_r, I_r^-(p',t) = I^-(p',t)$ and $I_r^+(p',t) = I^+(p',t)$
- $\forall h \in H, \forall f \in F,$

  $\forall p' \in P_r, I_r^-(p',hf) = I^-(p',h)$ and $I_r^+(p',hf) = I^+(p',h) + I^+(p',f) \circ I^-(p,f)^{-1} \circ I^+(p,h)$
- $\forall p' \in P_r, Mo_r(p') = Mo(p')$

<u>Example</u> (continued)



<u>Theorem</u> Let (R,Mo) be a coloured net and (R_r,Mo_r) be the reduced net by a post-agglomeration with multiple outputs , then the unfolded net of (R_r,Mo_r) is obtained by a sequence of pre-agglomerations starting from the unfolded net of (R,Mo).

<u>Proof</u>

The proof is decomposed in two parts. First we prove the theorem in the case where the orthonormal functions are identities. Next we prove that the general case can be reduced to the particular case.

Part A All u_h and v_f are identies.

Step 1 of part A

Let us verify that the transitions (f,c) with $f \in F$ are post-agglomerable :

- Their single input place is (p,c) and conversely the only output transitions of (p,c) are (f,c). This place is unmarked. The valuation of the arc between (p,c) and any (f,c) is 1.
- The input transitions of (p,c) are (h,<c,c'>) with $h \in H$ and $c' \in C_h$
- The valuation of the arc between (p,c) is 1.
- There is a transition f such that (f,c) has an output place.

Step 2 of part A

Let us verify that the reduction applied to $\{(f,c) \,/\, f \in F\}$ does not change the conditions of the post-agglomeration of any $\{(f,c') \,/\, f \in F\}$. It suffices to show that (p,c) and (p,c') do not share their neighbours and this is clear :

[{(f,c) / f ∈ F} U {(h,<c,c"> / h ∈ H and c" ∈ C_h }] ∩

[{(f,c') / f ∈ F } U {(h,<c',c"> / h ∈ H and c" ∈ C_h }] = ø

Hence one can successively apply all the post-agglomerations.

Step 3 of part A

Let us have a look at the reduced net .

- All the transitions (h,<c,c'>) and (f,c) have disappeared
- All the places (p,c) have disappeared
- New transitions (h.f,<c,c'>) have appeared
- Let $q \neq p$ such that q is not connected to H U F in the coloured net. Then for each $c \in C(q)$, all the arcs of (q,c) are unchanged.
- The input places of any (h.f,<c,c'>) are exactly the same as the input places of (h,<c,c'>) and have the same valuation.
- The valuation of the arc from (h.f,<c,c'>) to an output place (p',c") is :

$$I^+(p',h)\,(c'',<c,c'>) + I^+(p',f)\,(c'',c)$$

and since $I^+(p,h)$ is the projection function

($d \neq c \Rightarrow I^+(p,h)\,(d,<c,c'>) = 0$ and $I^+(p,h)\,(c,<c,c'>) = 1$), this valuation can be rewritten :

$$I^+(p',h)\,(c'',<c,c'>) + \sum_{d \in C(p)} I^+(p',f)\,(c'',d)\,.\,I^+(p,h)\,(d,<c,c'>) =$$

$$[\, I^+(p',h) + I^+(p',f) \circ I^+(p,h) \,]\,(c'',<c,c'>)$$

Then this reduced net is clearly the unfolded net of the coloured reduced net.

Part B First we reduce the coloured net by the successive u_h^{-1} orthonormalization of h where h ranges over H and v_f^{-1} orthonormalization of f where f ranges over F. Then the reduced net verifies the conditions of the part A. Once we have reduced this net by the reductions of part A, we again apply the u_h orthonormalization of hf where hf ranges over HxF and it is easy to see that the final net is the reduced coloured net ‰

Corollary Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by a coloured post-agglomeration with multiple outputs, π a main property. Then :

(R,Mo) verifies π <=> (R_r,Mo_r) verifies π

5.3 Post-agglomeration with a single output

In contrast to the post-agglomeration with multiple outputs, here F is reduced to a single transition. Then the coloured function which valuates an arc from a transition of H to the place p is less constrained : it must be an unitary function (a very weak condition). There are no more constraints on the colours domain of the transitions of H. The other conditions are the same as the post-agglomeration with multiple outputs.

Definition 1 **Post-agglomerable transitions - with a single output -**

Let (R,Mo) be a coloured Petri net, a transitions f is post-agglomerable if and only if there is a place p and a subset of transitions H with $H \cap \{f\} = ø$ such that the following conditions are fulfilled :

(1) $\forall t \in H$, $I^+(p,t) = 0$ and $\forall t \neq f$, $I^-(p,t) = 0$

$I^+(p,h) \neq 0$ and $I^+(p,h)$ is an unitary function

$C(f) = C(p)$ and $I^-(p,f)$ is an orthonormal function we call it v_f

$Mo(p) = 0$

(2) $\forall c \in C(p)$, $\exists p' \in P$, such that $I^+(p',f)(c) \neq 0$

(3) $\forall p' \neq p$, $I^-(p',f) = 0$

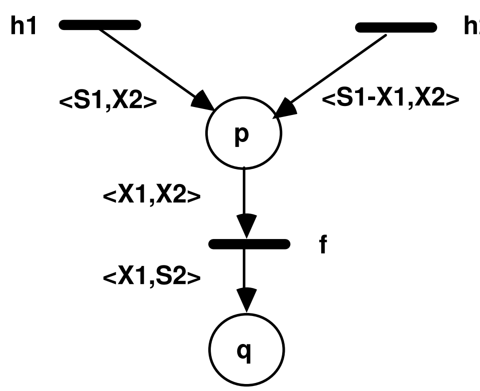
Remark In the second condition $I^+(p',f)(c)$ is an item of Bag(p') (Cf the notations)

Comparison If we compare our reduction rule with the reduction rule n° 2 given in [Col86], we can observe that our rule extends the rule n° 2 :

- In our rule, p may have several input transitions (the subset H) while in the other rule, only a single input transition is possible.
- In our rule, the coloured functions from H to p are unitary while in the other rule they are orthonormal. (orthonormal functions are unitary)

Example

C(h1) = C2 , C(h2) = C(p) = C(f) = C1 x C2 , C(q) = C1



In the post-agglomeration with a single output, the place p and the transition f disappear and one substitutes the "product" transitions of H x {f} for the transitions of H. The arcs of these transitions are obtained by the union of the arcs of H and {f) where the output arcs of f are successively composed by the inverse of the function valuating the arc between p and f and the function valuating the arc between h and p.

Definition 2 **Post-agglomeration of transitions with a single output**

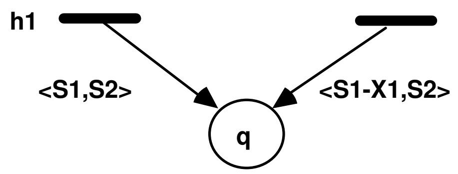
The reduced net (R_r,Mo_r) obtained from the net (R,Mo) by a coloured post-agglomeration of H and f is defined by :

- $P_r = P / \{p\}$
- $T_r = T / \{f\}$
- $\forall t \in T_r / H$, $\forall p' \in Pr$, $C_r(t) = C(t)$ and $C_r(p') = C(p')$
- $\forall t \in T_r / H$, $\forall p' \in P_r$, $I_r^-(p',t) = I^-(p',t)$ and $I_r^+(p',t) = I^+(p',t)$
- $\forall h \in H$, $\forall p' \in P_r$,

  $I_r^-(p',h) = I^-(p',h)$ and $I_r^+(p',h) = I^+(p',h) + I^+(p',f) \circ I^-(p,f)^{-1} \circ I^+(p,h)$
- $\forall p' \in P_r$, $Mo_r(p') = Mo(p')$

Example (continued) Notice here the composition of the functions are obtained by the symbolic substitution [Had87b] :

$<S_1,S_2> = <X_1,S_2> \circ <S_1,X_2>$ and $<S_1 - X_1,S_2> = <S_1 - X_1,S_2> \circ <S_1,X_2>$



Theorem Let (R,Mo) be a coloured net and (R_r,Mo_r) be the reduced net by a post-agglomeration with a single output, then the unfolded net of (R_r,Mo_r) is obtained by a sequence of pre-agglomerations starting from the unfolded net of (R,Mo).

Proof

The proof is decomposed in two parts. First we prove the theorem in the case where the orthonormal functions are identities. Next we prove that the general case can be reduced to the particular case.

Part A v_f is an identity function.

Step 1 of part A

Let us verify that the transition (f,c) is post-agglomerable :

- Its single input place is (p,c) and conversely the only output transition of (p,c) is (f,c). This place is unmarked. The valuation of the arc between (p,c) and any (f,c) is 1.
- The input transitions of (p,c) are (h,c') with $h \in H$ and $I^+(p,h)(c,c') \neq 0$
- The valuation of the arc between (p,c) and (h,c') is 1since $I^+(p,h)$ is unitary
- The condition (2) implies that (f,c) has an output place.

Step 2 of part A

Let us verify that the reduction applied to (f,c) does not change the conditions of the post-agglomeration of any (f,c'). Since there is a single transition in the set F' the new transitions of H' x F' may be identified as the transitions of H'.The places (p,c) and (p,c') may share their input transitions (some subset of {(h,c")}). But when the reduction is applied since (f,c) has not (p,c) for output place, the valuation of the arc between a transition (h,c") and the place (p,c') is unchanged.

Hence one can successively apply all the post-agglomerations.

Step 3 of part A

Let us have a look at the reduced net .

- All the transitions (f,c) and all the places (p,c) have disappeared
- Let $q \neq p$ such that q is not connected to H U {f} in the coloured net. Then for each $c \in C(q)$, all the arcs of (q,c) are unchanged.
- The input places of any (h,c) are unchanged.
- The valuation of the arc from (h,c) to an output place (p',c') is exactly:

$$I^+(p',h)\,(c',c) + \sum_{d \in C(p)} I^+(p',f)\,(c'',d)\,.\,I^+(p,h)\,(d,c) =$$

$$[\, I^+(p',h) + I^+(p',f) \circ I^+(p,h) \,]\,(c',c)$$

The sum over d is obtained by the successive reductions.

Then this reduced net is clearly the unfolded net of the coloured reduced net.

Part B

First we reduce the coloured net by the v_f^{-1} orthonormalization of f. Then the reduced net verifies the conditions of the part A. Once we have reduced this net by the reductions of part A, it is easy to see that the final net is the reduced coloured net ‰

Corollary Let (R_r,Mo_r) be a reduced net obtained from the net (R,Mo) by a coloured post-agglomeration with a single ouput, π a main property. Then :

$$(R,Mo) \text{ verifies } \pi \Longleftrightarrow (R_r,Mo_r) \text{ verifies } \pi$$

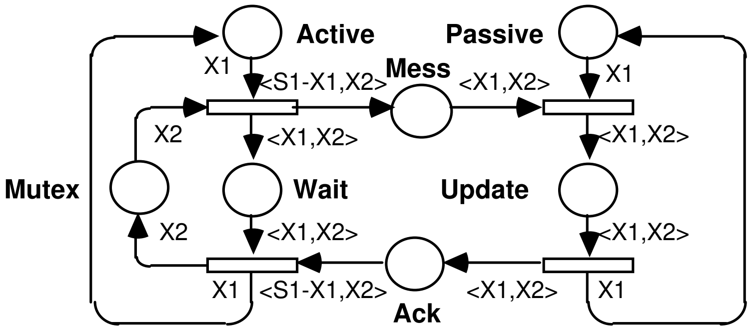
6. APPLICATION TO THE DATA BASE MANAGEMENT MODELLING

We present now the modelling of a data base management with multiple copies. This modelling is an improved version of those of [Jen81a]. Each site has two processes, an active one and a passive one. The access grant of a file of the data base is centralized and submitted to the mutual exclusion. In order to modify a file the active process of a site must get its grant and once it has modified the file, it sends messages to the others sites with the updated file. Then the passive processes update their own data base and send an acknowledgment. Once the active process has received all the acknowledgments, it releases the grant. Simultaneous accesses to different files are allowed.

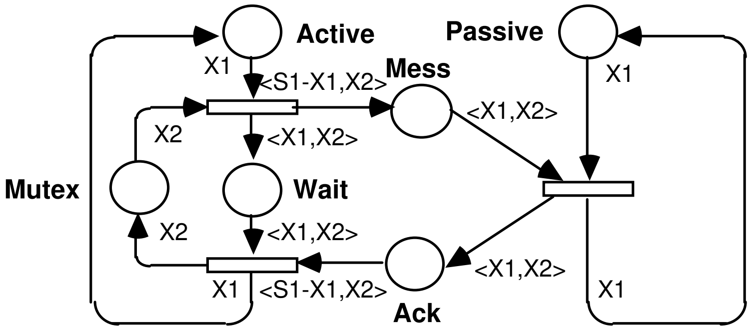
In the net, an active process must get in Mutex the single token coloured by the file it wants to access. The messages are composed by the name of the receiver followed by the name of the file. The acknowledgments are composed by the name of the sender followed by the name of the file. Accessing and modifying a file is modelled by a single transition (indivisible step) while the updating of the passive process is modelled by a place (divisible step). Initially there is a token per site in Active and Passive and a token per file in Mutex.

C1 = { Sites } , C2 = { Files } C(Active) = C(Passive) = C1 , C(Mutex) = C2
C(Wait) = C(Update) = C(Mess) = C(Ack) = C1 x C2

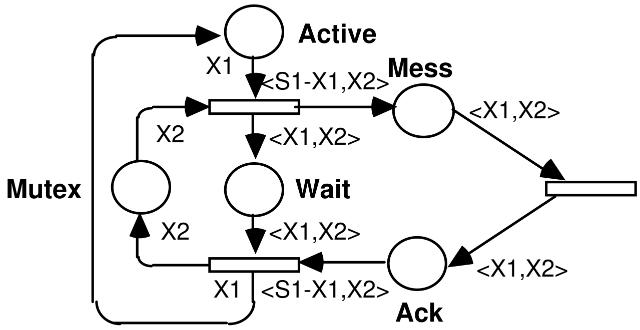
For every transition t , C(t) = C1 x C2, the coloured functions are defined as in the preceeding examples.



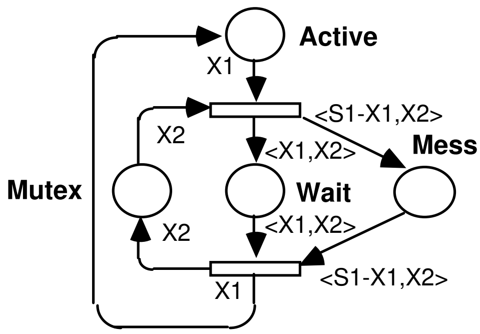
Post agglomeration with a single output around Update



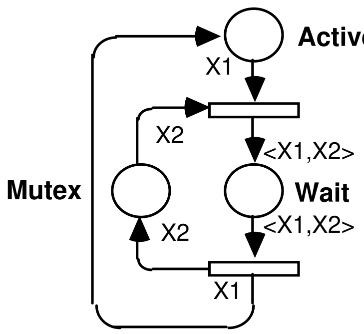
Simplification of the implicit place Passive



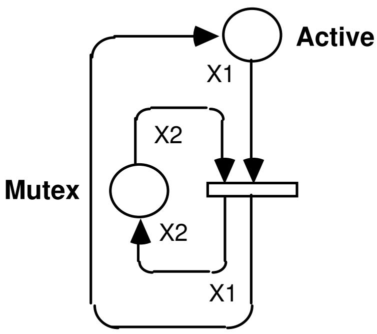
Pre agglomeration around Ack



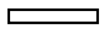
Simplification of the implicit place Mess



Post agglomeration
with a single output around Wait



Simplification of the implicit
pl aces Mutex and Active



In the final net (a single transition) all the main properties are verified. Thus the original net also verifies the main properties (boundness, liveness, …).

CONCLUSION

Here we have presented a methodology to generalize reductions for coloured nets. This methodology is based on two principles :

- Do not define, if possible, additional structural conditions for the extended reduction rules.
- Only define the functional conditions necessary to ensure the equivalence between the reduced net and the original net.

In order to illustrate this methodology we have extended the most frequently used rules of Berthelot. The two advantages of our reductions are:

- on the one side, they are strictly equivalent to the reductions defined by Berthelot and they then have the numerous properties proved by him;
- on the other side the functional conditions are not predefined but are the weakest possible necessary to obtain this equivalence in each case and then they have a large field of application.

With the coloured reductions, we have completely reduced an improved version [Had87b] of the data base management [Jen81a] with multiple copies. As we discussed in the beginning, reductions rules can still be improved. One way to do so, is to combine our reductions with the equivalence transformations of Genrich as we have already done with the orthonormalization of transitions improving our preceeding reductions [Had88]. Another way is to extend the fusions defined in [Ber86] , but it must be noticed that these reductions are based on behavioural conditions and then the extension problem is quite different.

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