Symbolic Reachability Graph
And Partial Symmetries

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Abstract

The construction of symbolic reachability graphs is a useful technique for reducing state explosion in High-level Petri nets. Such a reduction is obtained by exploiting the symmetries of the whole net [1]. In this paper, we extend this method to deal with partial symmetries. In a first time, we introduce an example which shows the interest and the principles of our method. Then we develop the general algorithm. Lastly we enumerate the properties of this Extended Symbolic Reachability Graph, including the reachability equivalence.

Topics: Analysis and synthesis, structure and behavior of nets.
Keywords:
Well-Formed Petri nets, Symbolic Reachability graphs, Partial Symmetries.

Introduction

Numerous verification techniques of systems have been defined from the theory of High-level Petri nets. The computation of reachability graphs is one of the most used, however, it has to cope with combinatorial explosion problem in space and time. Therefore we aim at reducing the size of the graph to be constructed, with regards to some desired properties. The symmetrical technique is one of the most successful since it gathers, into equivalence classes, the markings which cause the same behavior of the system [8][9]. Among the related methods, the symbolic reachability graph computation [1][2][4] has the following advantages: to be completely automatizable, to define classes of transition firings, and to be applied in stochastic analysis[3].

The former techniques exploit the existence of symmetries in the behavior of systems. However, they do not deal efficiently with what we call "partially symmetrical" systems. This is the case for a system, the behaviors of which sometimes depend on the process identities (e.g. static priorities based on identities), and sometimes not. With regards to such systems, the set of reachable symbolic markings are nothing more than the set of reachable ordinary markings. In this paper, we aim at extending the theoretical results brought out by [1] to take into account such partially symmetrical systems.

The method presented here can be summarized by the following points: (1) partition the nets, in particular the transitions, to a symmetrical part and an asymmetrical one; (2) optionally, add to the symbolic marking the relevant information in order to handle the firing of asymmetrical transitions; (3) re-define the symbolic firing rule for the "asymmetrical" part of the net; (4) build an Extended Symbolic Reachability Graph in such a way that the additional information is developed, only if it is required.

The next sections are organized as follows: part 2 briefly recalls the principles of the Symbolic Reachability Graph and highlights the limits of this approach through an example; part 3 formally defines the *Extended Symbolic Reachability Graph (ESRG)* and shows an algorithm for their constructions; part 4 brings out the main properties of the ESRG; part 5 is our conclusion.

2. Motivations and Informal Presentation

The starting point of our development is the model of Well-formed Petri Nets (WN) [1]. Such nets have the same expressive power as the common Colored Petri Nets [9], but define the color management in a very rigorous syntax. In particular, color domains and their associated functions are defined from *classes* of primitive objects and from *static subclasses* of these objects. Classes gather objects having the same nature, while static subclasses gather objects having the same behavior.

`Example`: *one may define the class Process ={p1,p2,p3} to model three processes, and may split Process into two static subclasses Interactive={p1,p2} and Batch={p3}.*

Roughly speaking, a standard symbolic marking is an equivalence class of ordinary markings, where the equivalence relation is deduced from a set of "*admissible permutations of colors*". The admissible permutations operate on classes. They preserve (1) the static subclasses and (2) the successor relation on class, when defined.

In the next section, a significant example is used to demonstrate the limits of this method, in case of partial symmetries. Then, the last paragraphs of this section present the construction principles of the "*Extended Symbolic Reachability Graphs*".

2.1. Example : a Critical Section

Let consider a set of processes which may access to a critical section. A process is in one of the following four states: "idle", "wait", "global select" or “CS” (i.e. critical section).

Each process may send a request for the access of a critical section and each one may know the other requests. Hence, each process evaluates independently which one will have the access. The conflict management is based on the identities of processes such as the largest identification number is always the most important. When a process accesses to the critical section, the other ones cannot issue a request.

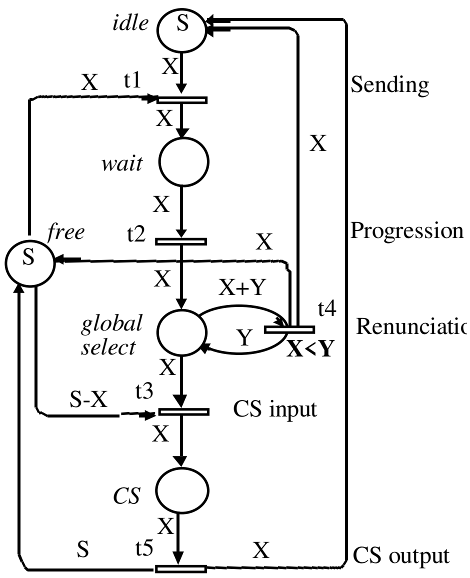

Figure 2.1: critical section

The Well-formed Petri net of Figure 2.1 models the management of requests. The WN formal definition is recalled in annex.

Let $C_1=\{<1>,<2>,<3>\}$ *a class of three processes. Let* $C_1^1=\{<1>\}$, $C_1^2=\{<2>\}$ *and* $C_1^3=\{<3>\}$ *be the three elementary static subclasses of* C_1 *(elementary means that each subclass has only one element).*

Moreover, X and Y are projections of codomain C_1*; S-X allows the testing of all the objects of* C_1 *, except of the one which is represented by X.*

For sake of clarity, we limit the number of nodes in the reachability graph, by assuming that t3 and t4 transitions have a higher priority order than that of t1.

The "idle" state is the initial state for all the processes. The conflict management is modeled by three (shared) places: the "free" place is used to store the identities of the processes which have not sent a request, the "wait" place models the current requests in terms of process identities; the "global select" place is used to solve conflicts. The access to the critical section is modeled by the "CS" place. Initially, the "free" place is set to all the colors of processes meaning that there is no request.

The t1 transition models the sending of a request from any X process, therefore its firing removes an X color from the "idle" and "free" places and adds the same color into the "wait" place. The t2 transition models the fact that a request becomes known by the whole set of processes: its firing removes an X color from the "wait" place and adds it into the "global select" place. The conflict management is modeled by two transitions: t3 and t4. In case of several colors in the "global select" place, only the t4 transition is firable; its goal is to compare two process identities of the "global select" place such as the color having the largest number is kept, while the other changes to update the "free" and "idle" places. When there is only one token in the "global select" place and all the other colors are in the "free" place, only the t3 transition is firable. This last transition models the access to the critical section by the selected X process, therefore its firing removes an X color from the "global select" place, removes the other ones from the "free" places and adds the X color into the "CS" place. The t5 transition models the end of the access, therefore its firing resets the "free" and "idle" places to their initial marking.

It is worth noting that all the processes have symmetrical behaviors, except when the t4 transition become firable. Indeed, this transition is the only one, the firing of which depends on the process identities. Let us compute the set of "admissible permutations of colors", with regards to the C1 class and the guard of the t4 transition:

The t4 guard is defined by guard[t4]: $C_1 \times C_1 \rightarrow \{T, F\}$

$(x,y) \rightarrow x < y$

Let s be a permutation of the C_1's colors. Accordingly to [8],

the s permutation is enabled if and only if guard[t4]o<s,s>= guard[t4].

But: guard[t4] o <s,s>: $(x,y) \rightarrow s(x) < s(y)$

and there is only one permutation which preserves an ordering over a set: the identity function.

We conclude that the set of admissible permutations for the C1 class is reduced to the identity function. This leads us to define as many static subclasses as the C1 cardinality is. Such a splitting is very penalizing because each equivalence class will have only one element. Hence, the resulting standard symbolic reachability graph will have the same size as the ordinary reachability graph. This drawback is common to all the current symmetrical techniques [1][8].

In this paper, we propose to cope with this problem by distinguishing the "asymmetrical" part from the remainder of the net. Such an idea seems to be interesting with regards to our claiming that very often the symmetrical part of a model is larger than the asymmetrical one. We now introduce the notion of *Extended Symbolic Reachability Graph* (ESRG).

2.2. Presentation of Extended Symbolic Reachability Graphs

Our method is based on the structural detection of "*distinguished classes*", which are classes for which asymmetrical behaviors occur. In the following, only one distinguished class is considered since the extension to several of these classes do not present any theoretical difficulty. Moreover, we assume that the distinguished class is is partition in as many static subclasses as the number of objects in the class.

In order to present our method, we now follow the following four stages:
- the partitioning of the net in a symmetrical part and an asymmetrical part,
- the representation of extended symbolic markings,
- the representation of the extended symbolic firing rule,
- the technique to build Extended Symbolic Reachability Graphs.

2.2.1 Partition the Transitions

The use of the operators: "<", "≤", ">" and "≥" cause asymmetrical behaviors, since they need to distinguish the objects of the tested classes. In terms of well-formed nets, such operators are expressed with membership tests, according to static subclasses. So the asymmetrical property of the Well-formed Nets can be featured by the specification of expressions, namely asymmetrical expressions, having membership tests.

In WN, the instances of variables are local to transitions, therefore, we use the term of *asymmetrical variable with respect to a transition*. This means that the considered variable is used in an asymmetrical expression, either in predicate functions associated with the arcs adjacent to the transition or in the transition's guard. Such a transition is called an *asymmetrical transition*. A transition which is not asymmetric is called a *symmetrical transition*. The partition of the set of transitions will allow us to consider two subnets, sharing the same places: the *asymmetrical subnet* and the *symmetrical subnet*. The two subnets differ from the type of their transitions, either symmetrical or asymmetrical, and from the fact that a specific symbolic firing rule is associated with each of them (see section 2.2.3).

Example of transition partition

In the net of Figure 2.1, X and Y are asymmetrical variables, with respect to the guard of t4 and its "<" operator. The t1, t2, t3 and t5 are symmetrical transitions.

2.2.2 Extended Symbolic Markings

An extended symbolic marking (ESM) may be viewed as a *symbolic marking*, optionally associated with a set of *eventualities*.

A standard symbolic marking is a marking, the colors of which are gathered into equivalence classes, forgetting the identity of colors but keeping the cardinality of each represented equivalence class. Such collections are defined from the notion of *dynamic subclasses*, which are dynamic decompositions of static subclasses of objects, in order to take into account the distribution of objects between places. In this approach, any marking is defined in terms of tuples made of dynamic subclasses, but no more in terms of colors. The formal definition of symbolic markings may be found in [1][4].

A set of eventualities are the set of possible partial instances, with respect to a standard symbolic marking and to a distinguished class. Hence, one has the ability to represent the behaviors of partially symmetrical systems.

Example

Figure 2.2 represents an example of extended symbolic marking, which corresponds to the WN of Figure 2.1. It is composed of two parts:

Figure 2.2: An extended symbolic marking

(idle,wait,g.select,CS,free)
(Z_1 , 0 , Z_2 , 0, Z_1)
$\lvert Z_1 \rvert=1$ $\lvert Z_2 \rvert=2$
E_1 $Z_1=\{3\}$ $Z_2=\{1,2\}$
E_2 $Z_1=\{2\}$ $Z_2=\{1,3\}$
E_3 $Z_1=\{1\}$ $Z_2=\{2,3\}$

• *The grey part corresponds to the classical representation of a symbolic marking. The C1 class is split into two dynamic subclasses, Z_1 and Z_2 , of* cardinality 1 and 2 respectively. *This means that two processes have sent a request (see "global select") while the third one is in the idle state (see the "idle" and "free"). Thus, there is only one process that has not sent a request.*

• *The white part of the extended symbolic marking corresponds to the three associated eventualities: E1, E2 and E3. The Z_1 and Z_2 dynamic subclasses are instantiated, since the C1 class is a distinguished class. This leads to three representations E1, E2 and E3, called eventualities.*

It is worth noting that eventualities are not markings, but partial instances of ESMs. However, one of our aim is to represent them only when it is required. In fact, the necessity to represent them is conducted by the fact that two cases may occur, with respect to an extended symbolic marking,:

(1) some asymmetrical transitions are enabled from at least one of the eventualities;

(2) some of the eventualities are not reachable.

In order to decide of the construction of ESM eventualities, we now define the notion of *saturation and uniformity* which are checked during the construction of each ESM:

Definition 2.1.: Saturated and Uniform ESMs

An ESM is said to be saturated if and only if all its eventualities are reachable; it is said to be uniform if and only if the objects of the distinguished class are gathered in the same place.

The following property highlights two cases for which the eventuality representation of ESMs can be useless. In this paper and for reasons of clarity, the initial ESM is assumed to be uniform.

Property 2.1.:

An ESM can be represented by its underlying symbolic marking if one of the two following conditions hold:
(1) an ESM is saturated and there is no asymmetrical transition enabled from it ,
(2) an ESM is uniform.

Effectively, in the first condition the whole set of eventualities of the ESM are reachable and enabled by the same symbolic instances of (symmetrical) transitions, hence, the underlying symbolic marking represents the ESM completely; in the second case, all the elements of the distinguished class are gathered in the same dynamic subclass, leading to reduce the set of eventualities to only one item.

The following property, which is directly inherited from the SMG theory, will be used in the general algorithm for the construction of ESRGs.

Property 2.2.: Propagation of Saturation

An ESM which is reached from a saturated ESM, by means of a symmetrical transition firing, is also saturated.

2.2.3 Extended Symbolic Firing Rules

As for standard symbolic markings, the construction of an extended symbolic marking can be performed without computing the underlying reachable marking. The condition is that the extended symbolic firing rule takes an ESM into account, in order to build the resulting ESM representations directly.

Our method to define a suitable symbolic firing rule is based on the idea that the subclasses of the distinguished class must be introduced only to deal with asymmetrical transitions. Thus, we choose to build the standard symbolic marking of any ESM without considering the decomposition of the distinguished class in static subclasses, however, when necessary, another standard symbolic representation can be computed from it by considering any given eventuality. Hence, the enabling test can always be performed from a standard symbolic representation of ESMs,

The extended symbolic firing rule will be used to build a graph in which the nodes are the reachable ESMs . The extended firing rule will be used to define the arcs linking those nodes. From a graph point of view, an ESM must be viewed as a set of nodes: one for each standard symbolic marking or eventuality. Thus, there are four possibilities of links between ESMs: from a SM to another SM, from an E to a SM, from an E to another E and from an SM to an E.

Three types of rules are defined:

• the *generic symmetrical firing* occurs for symmetrical transitions, if the extended symbolic marking to take into account is saturated. This case directly relates from the standard symbolic firing; the source and the target are standard symbolic markings;

• the *instantiated symmetrical firing* occurs also for symmetrical transitions, but in case of an absence of saturation. In this case, the source is a reachable eventuality, while the target is either an eventuality or a standard symbolic marking , depending on the ability to make the eventualities absent or not. Due to the symmetrical property of the transition, any reachable eventuality has the same ability of firings and reaches the same extended symbolic markings.

• the (instantiated) *asymmetrical firing* occurs for asymmetrical transitions; here again, the source is a reachable eventuality, while the target is either an eventuality or a standard symbolic marking. Due to the asymmetrical property of the transition, the eventualities of an ESM may not have the same ability of firings (mixed existence of dead and live eventualities, target nodes may be different...).

Table 2.4 summarizes the types of firings according to types of transitions and marking conditions.

type of transitions *marking conditions*	*symmetrical*	*asymmetrical*
saturated	*generic symmetrical*	*(instantiated) asymmetrical*
not saturated	*instantiated symmetrical*	*(instantiated) asymmetrical*

Table 2.4: Use of Firing Types

`Example`

In Figure 2.5, $\mathcal{M}e_4$, $\mathcal{M}e_5$ and $\mathcal{M}e_{10}$ are assumed to be reachable ESMs of the net of Figure 2.1. $\mathcal{M}e_4$ and $\mathcal{M}e_5$ are assumed to be saturated, therefore their eventualities are not represented.

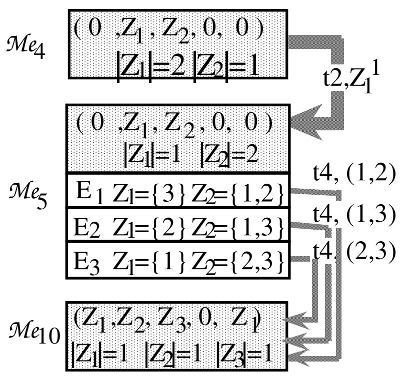

Figure 2.5:
Example of Extended Symbolic Firings

From $\mathcal{M}e_4$, the t2 symmetrical transition is enabled. Since there is no asymmetrical transition enabled from $\mathcal{M}e_4$, a generic symmetrical firing can occur by t2. Hence, the firing of t2 takes into account one item of Z_1, isolated in the Z_1^1 dynamic subclass. From $\mathcal{M}e_5$, the t4 asymmetrical transition is enabled, therefore the $\mathcal{M}e_5$'s eventualities must be considered. Each of these eventualities is the source of a firing of t4: from E1: X=<1> and Y=<2>, from E2: X=<1> and Y=<3>, from E3: X=<2> and Y=<3>.

2.2.4. Extended Symbolic Reachability Graph

The definitions of extended symbolic markings and extended symbolic firing rules allow us to build Extended Symbolic Reachability Graphs (see an efficient algorithm in section 3.4).

`Example`

Figure 2.6 represents the extended symbolic graph of the model depicted in Figure 2.1. There are two types of arcs: *symbolic arcs* (see bold arcs) link two symbolic markings, while *instantiated arcs* (see standard arcs) link an eventuality of an extended symbolic marking to another node.

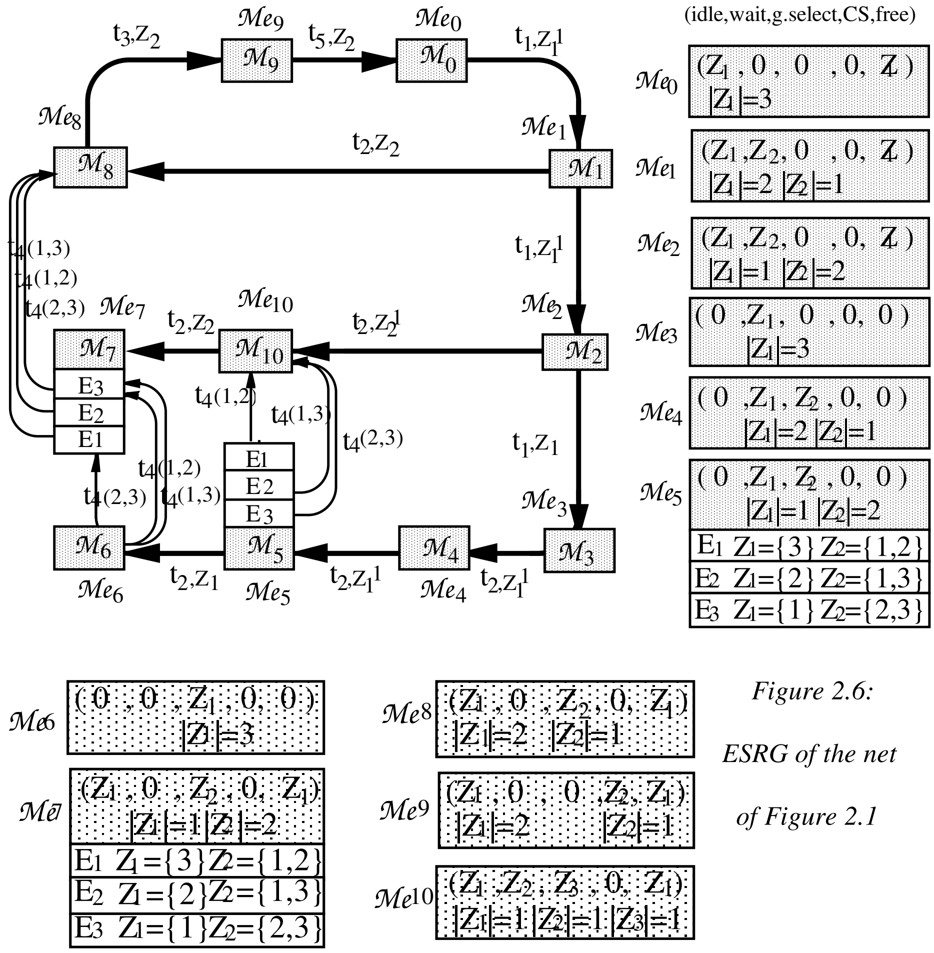

Figure 2.6: ESRG of the net of Figure 2.1

There are 11 nodes in this graph while the corresponding Symbolic Reachability Graph contains 30 markings. In this graph, all the extended symbolic markings are saturated. Indeed, each one is the target of a saturated symbolic node. Only, the $\mathcal{M}e_5$, $\mathcal{M}e_6$ and $\mathcal{M}e_7$ extended symbolic markings make the t4 transition firable, therefore all the arcs are symbolic, except of $\mathcal{M}e_5$ to $\mathcal{M}e_{10}$, $\mathcal{M}e_6$ to $\mathcal{M}e_7$ and $\mathcal{M}e_7$ to $\mathcal{M}e_8$, which are instantiated arcs. Moreover, on can note that $\mathcal{M}e_0$, $\mathcal{M}e_3$ and $\mathcal{M}e_6$ are uniform. Since it is the case for $\mathcal{M}e_6$, only the eventualities of $\mathcal{M}e_5$ and $\mathcal{M}e_7$ have to be represented.

3. Extended Symbolic Reachability Graph

In this part, we define formally the stages for the construction of an Extended Symbolic Reachability Graph.

3.1. Partition of Well-formed Nets

The formal definition of Well-formed Nets is recalled in appendix. We first define the notion of "asymmetrical variables", which are variables used in membership tests. This leads to define the notions of "asymmetrical transition" and "asymmetrical subnet" of the WN. The other part of the net and its transitions are named "symmetrical". Let us recall that the decomposition in static subclasses for the distinguished class is not considered in the symmetrical subnet.

Definition 3.1 Asymmetrical Variable With Respect to a Predicate Function or a Guard

Let C_d be the distinguished class.

A variable X defined on C_d is said to be asymmetrical if and only if there exists a predicate function or a guard such that one of the two following conditions hold:

(i) the belonging of X to any static subclass of Cd is tested,

(ii) X is in relation with an asymmetrical variable, by the use of one of the following Well-formed Net's operator: $=, \neq, \oplus$.

In the following, such predicate function or guard are said to be asymmetrical.

Definition 3.2 Asymmetrical and Symmetrical transitions

Let t be a transition of a Well-formed Net

t is said to be asymmetrical if and only if one of the three following conditions hold:

(i) there is a place p of P such that there is an asymmetrical predicate function in $W^-(p,t)$ or in $W^+(p,t)$.

(ii) the guard of t is asymmetrical .

(iii) there is a place p of P such that there is a diffusion function in $W^-(p,t)$ or in $W^+(p,t)$, defined on the distinguished class.

t is said to be symmetrical if and only if t is not asymmetrical .

Definition 3.3 Asymmetrical Subnet and Symmetrical Subnet of WN

Let $WN = \langle P, T_{asym} \cup T_{sym}, C, W^-, W^+, \phi, \pi, M_0 \rangle$.

- The symmetrical subnet of WN is the $\langle P, T_{sym}, C, W^-, W^+, \phi, \pi, M_0 \rangle$ tuple, in which the partition of WN classes in static subclasses is preserved, except for C_d. In that subnet, C_d is considered without static subclass decomposition.

- The asymmetrical subnet of WN is the $\langle P, T_{asym}, C, W^-, W^+, \phi, \pi, M_0 \rangle$ tuple, in which the partition of WN classes in static subclasses is preserved.

3.2. Extended Symbolic Markings

An extended symbolic marking is an equivalence class of markings. Like symbolic marking, the definition of equivalence is based on the constraint of "admissible permutations of objects", meaning that permutations must preserve the belonging to static subclasses". So, two markings are equivalent if and only if they are equal, according to any admissible permutation. Unlike standard symbolic marking, this constraint does not concern the distinguished classes, inducing the definition of bigger equivalence classes. One may report to [1][4] in order to have a formal definition of such equivalence relation.

The representation of an extended symbolic marking is composed of two parts: a representation close from this of standard symbolic marking and optionally, a set of "eventualities" which are defined as instances of the standard symbolic marking over the distinguished class.

Definition 3.4 Representation of an Extended Symbolic Marking

Let $\mathcal{Me}$ be an extended symbolic marking.

A representation of $\mathcal{Me}$ is a pair: $\mathcal{Re} = <\mathcal{R}, \mathcal{E}>$ where,

(i) $\mathcal{R}= <m,card,d,marq>$ is the representation of a standard symbolic marking;

let us recall that:

- m maps each class in an index of "dynamic subclasses". Dynamic subclasses partition the objects of any C_i class according to a distribution of these objects in places, with respect to a marking. Their names are built as follows: Z_i^j denotes the j^{th} of the C_i class. Moreover, the set of the dynamic subclasses of C_d is denoted:

$\widehat{C}_d = \{Z_i^j: 0<j\leq m(d)\}$.

- card features the number of objects represented by each dynamic subclass.
- d maps each dynamic subclass in a number qualifying the corresponding static subclass (if there is some).
- marq maps each place in a tuple of dynamic subclasses;

(ii) $\mathcal{E} = \{ E : C_d \rightarrow \widehat{C}_d \ / \ |E^{-1}(Z_d^j)| = card\ (Z_d^j) \}$ or $\mathcal{E} = \varnothing$

In the following:

An extended symbolic marking is denoted by a pair $\mathcal{Me}=<\mathcal{M},\mathcal{E}>$ where $\mathcal{M}$ is the underlying standard symbolic marking and $\mathcal{E}$ is the associated set of eventualities. Moreover, $\mathcal{Me.M}$, $\mathcal{Me.E}$ denotes the components of $\mathcal{Me}$. Similarly, $\mathcal{R}$.m, $\mathcal{R}$.card, $\mathcal{R}$.d, $\mathcal{R}$.marq denotes the components of $\mathcal{R}$.

One of the major property of standard symbolic markings is that a canonical expression can be defined for each one, allowing easy comparisons. A canonical expression can be also defined for an extended symbolic marking:

Property 3.1 Canonical Representation

Let $\mathcal{Me}=<\mathcal{M},\mathcal{E}>$ be an extended symbolic marking.

$\mathcal{Re} = <\mathcal{R}, \mathcal{E}>$ is the canonical representation of $\mathcal{Me}$ if and only if $\mathcal{R}$ is the canonical expression of $\mathcal{M}$.

The proof is obvious since $\mathcal{E}$ is either empty or may be defined from $\mathcal{M}$, without ambiguity. As a consequence, the comparison of two ESMs is performed directly on their corresponding standard symbolic markings.

3.3. Extended Symbolic Firing Rules

We now show the extensions of the standard symbolic firing rule, with respect to the symmetrical subnet and to the asymmetrical one of a Well-formed Net. All the symbolic firing definitions are based on the notion of symbolic instances which is recalled in appendix. Let I be the set of class indexes and J be the set of class indexes except of "d".

Like the standard symbolic firing rule, all the types of firings are performed in four stages from an extended symbolic marking: the first stage is the splitting of the marking representation, in order to isolate any combination of symbolic colors that may be used for a firing, the second stage consists in the effective firings, with respect to symbolic colors of transitions, yielding for each one an extended symbolic marking; during the third stage, a minimal representation is obtained, by grouping dynamical subclasses of a same class if they have the same distribution on places; the fourth stage computes the canonical representations for each resulting marking.

A generic symmetrical firing occurs from a standard symbolic marking to another one, therefore its definition refers to the standard symbolic firing rule. An example of such classical firing can be extracted from Figure 3.1 by only considering the underlying symbolic markings of represented ESMs and bold arcs.

Definition 3.5 Generic Symmetrical Firing

Let $\mathcal{Me}=<\mathcal{M},\mathcal{E}>$ and $\mathcal{Me}'=<\mathcal{M}',\mathcal{E}'>$ be two extended symbolic markings.

Let t be a symmetrical transition such that : $C(t)=\prod_{i\in Bag(I)} C_{\alpha(i)}$,

We say that $\mathcal{Me}'.\mathcal{M}'$ is reached from $\mathcal{Me}.\mathcal{M}$ by the firing of t for the $(\prod Z_{\alpha(i)}^{\lambda(i),\mu(i)})$ symbolic instance ($i\in Bag(I)$), if and only if

$\mathcal{Me}.\mathcal{M}[\,(t, \prod Z_{\alpha(i)}^{\lambda(i),\mu(i)})>\mathcal{Me}'.\mathcal{M}'$ is a standard symbolic firing.

We denote this extended symbolic firing by : $\mathcal{Me}.\mathcal{M}\,[(t,\hat{c})>\mathcal{Mec}'.\mathcal{M}'$,

where $\mathcal{Mec}'$ is the canonical representation of $\mathcal{Me}'$,

and where $\hat{c}$ is a product of dynamic subclasses.

The following definition concerns the instantiated symmetrical firing. It occurs from an eventuality of an ESM and reaches an eventuality of another ESM. Despite the fact that the static subclasses of C_d are not considered in the symmetrical subnet, such type of firing deals with eventualities, causing us to isolate the dynamic subclasses of C_d during the firing.

Definition 3.6 Instantiated Symmetrical Firing

Let $\mathcal{Me}=<\mathcal{M},\mathcal{E}>$ and $\mathcal{Me}'=<\mathcal{M}',\mathcal{E}'>$ be two extended symbolic markings.

Let E (resp. E') be an eventuality of $\mathcal{Me}.\mathcal{E}$ (resp. $\mathcal{Me}'.\mathcal{E}'$).

Let t be a symmetrical transition such that: $C(t)=\prod C_{\alpha(j)} \times (C_d)^n$, ($j\in Bag(J)$).

We say that $\mathcal{Me}'$.E' is reached from $\mathcal{Me}$.E by the firing of t for the $(\prod Z_{\alpha(j)}^{\lambda(j),\mu(j)} \times \prod c_{\delta(k)})$ instance ($j\in Bag(J)$, $k\in 1..n$), if and only if the four following points hold :

(1) $c_{\delta(k)} \in C_d$,

(2) $\mathcal{M}[\,(t, \prod Z_{\alpha(j)}^{\lambda(j),\mu(j)} \times \prod Z_d^{\lambda(k),\beta(k)})>\mathcal{M}'$,

(3) $E(c_{\delta(k)}) = Z_d^{\lambda(k),\beta(k)}$

(4) $c_{\delta(k)} = c_{\delta(k')}$ if and only if $\beta(k)=\beta(k')$.

We denote this extended symbolic firing by : $\mathcal{Me}$.E $[(t,\hat{c})>\mathcal{Mec}'.\{Ec'\}$,

where $\mathcal{Mec}'$ and Ec' are the canonical representations of $\mathcal{Me}'$ and E',

and where $\hat{c}$ is a product of dynamic subclasses.

Comments :

(3) means that the $\mathcal{M}'$ standard symbolic marking is reached from $\mathcal{M}$ by the standard symbolic firing of the t transition for the $(\Pi Z_{\alpha(j)}{}^{\lambda(j),\mu(j)}$ x $\Pi Z_d{}^{\lambda(k),\beta(k)})$ instance; (4) means that the k^{th} instance on the C_d class corresponds to the k^{th} instanciated dynamic subclass of C_d; (5) means that some of the C_d's instances may correspond to the same color.

The operational scheme for the former definition needs an algorithmic expression which is not reported in this paper. However, we explain it through the following example.

`Example:`

Let us consider the net a Figure 3.1 with its t1 symmetrical transition. Let us consider the $\mathcal{M}e$ current marking of Figure 3.2. Let us assume that only the E1 eventuality is reachable from the initial extended symbolic marking ($\mathcal{M}e$ is not saturated).

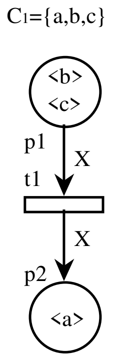

Figure 3.1 A simple net

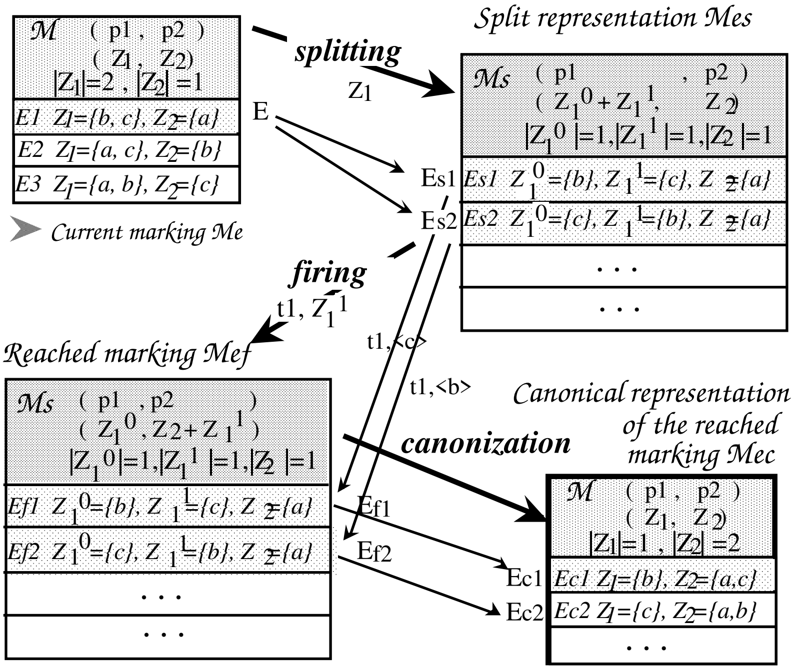

Figure 3;2: stages of an instantiated symmetrical firing

`Comments:` *To the E1 eventuality, corresponds the two* E_{s1} *and* E_{s2} *eventualities in the* $\mathcal{M}es$ *split representation. Since the t1 symmetrical transition is enabled from* $\mathcal{M}es.\mathcal{M}s$ *for the* Z_1^1 *symbolic instance, it is also enabled from any eventuality of* $\mathcal{M}es.\mathcal{E}$*, in particular* E_{s1} *and* E_{s2}*.* Z_1^1 *refers respectively to {c} in* E_{s1} *and to {b} in* E_{s2}*. So, the canonical* $\mathcal{M}ec$ *extended symbolic marking may be reached from* $\mathcal{M}e$*, by one of the two (instantiated) symmetrical firings of t1, one for the instance <b> and the other for <c>.*

We now present the firing rule for an asymmetrical transition. It occurs from an eventuality of an ESM to another eventuality of an ESM. The static subclasses of the distinguished class are taken into account to test the transition, therefore, we introduce first the notion of split marking with respect to an ESM.

Definition 3.7 Split marking wrt. an eventuality

Let $\mathcal{M}e$=<$\mathcal{M}$,$\mathcal{E}$> be an extended symbolic marking.
Let E be an eventuality of $\mathcal{M}e.\mathcal{E}$.
$\mathcal{M}$ is a symbolic marking of the symmetrical net, therefore $\mathcal{M}$ may be transformed by partitioning the static subclasses of C_d in dynamic subclasses (one for each color).
$\mathcal{M}_E$ is called the split representation of $\mathcal{M}e.\mathcal{M}$ wrt. E.

Definition 3.8 Asymmetrical firing

Let $\mathcal{M}e$=<$\mathcal{M}$,$\mathcal{E}$> and $\mathcal{M}e'$=<$\mathcal{M}'$,$\mathcal{E}'$> be two extended symbolic markings.
Let E (resp. E') be an eventuality of $\mathcal{M}e.\mathcal{E}$ (resp. $\mathcal{M}e'.\mathcal{E}$).
Let t be an asymmetrical transition such that $C(t)=\Pi\, C_{\alpha(i)}$,
We say that $\mathcal{M}e'$.E' is reached from $\mathcal{M}e$.E by the firing of t for the $(\Pi Z_{\alpha(j)}{}^{\lambda(j),\mu(j)} \times \Pi C_d{}^{\delta(k)})$ symbolic instance, if and only if the three following points hold :

(1) $i \in Bag(I)$, $j \in Bag(J)$
(2) $\mathcal{M}_E[\, (t, \Pi Z_{\alpha(i)}{}^{\lambda(i),\mu(i)} \times \Pi C_d{}^{\delta(k)}) > \mathcal{M}'_{E'c}$,
(3) $\mathcal{M}'_{E'c}$ is the canonical representation of $\mathcal{M}'_{E'}$.

<u>We denote</u> this extended symbolic firing by : $\mathcal{M}e.E\ [(t,\hat{c})> \mathcal{M}ec'.E_c'$,
where $\mathcal{M}ec'$ and E_c' are the canonical representations of $\mathcal{M}e'$ and E',
and $\hat{c}$ is a product of dynamic subclasses.

The asymmetrical firing stages are similar to these of the instantiated symmetrical firing. Therefore one may refer to Figure 5.2 in order to have an example.

3.4. Construction of Extended Symbolic Reachability Graph

Our algorithm to build a standard symbolic reachability graph consists of computing the resulting enabled firings and the resulting symbolic marking, from any reachable symbolic marking. The implicit stack of a recursive function call is used to store the computed SMs before analyzing them. The canonical representation of symbolic markings allows one to decide whether a computed SM has been already computed. Such algorithm ends when all the different computed SM are analyzed.

The construction of an ESRG with the same strategy may cause redundancies of reachability. Effectively, an instantiated symmetrical firing may be computed before having the ability to produce a generic symmetrical firing, covering it. To cope with this problem, we propose to privilege generic symmetrical firings with respect to the other kinds of firings. Our technique consists of handling an explicit stack of ESM to store any new ESM. The implicit stack of the recursion is used to store ESMs once the firings of corresponding symmetrical transitions are achieved.

The algorithm is the following, with respect to a global variable G, representing the computed graph:

Compute_ESRG($\mathcal{M}e_0$)

Put $\mathcal{M}e_0$ in G

Develop_ESM($\mathcal{M}e_0$)

Remove_unreachable_E(G)

End_Compute_ESRG.

Comments:

$\mathcal{M}e_0$ is put in G, then the "Develop_ESM" function develops the graph. The Remove_unreachable_E *functions removes the unreachable eventualities of the computed ESM. Indeed, one cannot decide of the reachability of eventualities during the construction of an ESM, with some exceptions due to saturation or uniformity properties.*

Develop_ESM($\mathcal{M}e$)

for_all t such that symmetrical(t) For_all $(t,\hat{c})$ such_that $\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$

begin if saturated($\mathcal{M}e$) or uniform($\mathcal{M}e$)

then begin add_in_G ($\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$)

if saturated($\mathcal{M}e$) then saturated($\mathcal{M}e'$) end

else if uniform($\mathcal{M}e'$) then for_all E of s add_in_G ($\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$)

else add_in (G, $\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\{E'\}$)

if new_in_G($\mathcal{M}e'$) then push_in_stack($\mathcal{M}e'$)

end (for_all)

while not empty_stack() do begin pop_from_stack($\mathcal{M}e'$)

Develop_ESM($\mathcal{M}e'$)

end (while)

For_all asymmetrical(t) For_all $(t,\hat{c})$ such_that $\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$

begin if uniform($\mathcal{M}e$) then if uniform($\mathcal{M}e'$)

then begin add_in_G($\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$)

if saturated($\mathcal{M}e$) then saturated($\mathcal{M}e'$) end

else add_in_G($\mathcal{M}e.\mathcal{M} \xrightarrow{(t,\hat{c})} \mathcal{M}e'.E'$)

else if uniform($\mathcal{M}e'$) then begin add_in_G($\mathcal{M}e\,E \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$)

if saturated($\mathcal{M}e$) then saturated($\mathcal{M}e'$) end

else add_in_G($\mathcal{M}e\,E \xrightarrow{(t,\hat{c})} \mathcal{M}e'E'$)

if new_in_G($\mathcal{M}e'$) then Develop_ESM($\mathcal{M}e'$)

end(For_all)

if saturated($\mathcal{M}e$) and if no t such_that enabled($\mathcal{M}e$,t) then

for_all $\mathcal{M}e\,E$ such_that no $\mathcal{M}e\,E \xrightarrow{(t,\hat{c})} \mathcal{M}e'.\mathcal{M}'$ do remove_node(E)

End_Develop_ESM

Comments: "Develop_ESM" is a recursive function, dealing first with enabled symmetrical transitions. Any new enabled firing is added in G as well as the resulting marking if it is a new one. Any new marking is stored in the explicit stack. Before dealing with enabled asymmetrical firings, an explicit stack is emptied to search markings having other symmetrical firings. Lastly, if there is no enabled asymmetrical transition from a current ESM, The ESM's eventualities are removed.

In order to prepare the Remove_unreachable_E call, an input arc counter is associated with any eventuality or uniform ESM, is updated during the Develop_ESM call. With the assumption that the initial extended symbolic marking is uniform, we can associate with a counter, the value of which is 1.

Remove_unreachable_E()
while_there_exists $\mathcal{S}$ such_that cnt($\mathcal{S}$)=0 do
remove_node($\mathcal{S}$)
For_all $\mathcal{S}'$ such_that $\mathcal{S} \to \mathcal{S}'$ decrement_cnt($\mathcal{S}$)
if no_eventuality($\mathcal{S}$) then remove_ESM($\mathcal{S}$)
End_Remove_unreachable_E

Example
Let us perform the ESRG of Figure 2.6:
From the $\mathcal{M}e_0$ initial extended symbolic marking, the 1 to 8 firings allow the construction of the six following ESMs: $\mathcal{M}e_1$, $\mathcal{M}e_8$, $\mathcal{M}e_2$, $\mathcal{M}e_{10}$, $\mathcal{M}e_3$, $\mathcal{M}e_4$, $\mathcal{M}e_5$ and $\mathcal{M}e_6$. All these firings are of generic symmetrical type. Since $\mathcal{M}e_6$ does not lead to any symmetrical firing, the computation deals with $\mathcal{M}e_{10}$. Number 9 firing is then performed, leading to $\mathcal{M}e_7$. Since $\mathcal{M}e_7$ does not lead to any symmetrical firing, the computation deals with $\mathcal{M}e_8$. Number 10 firing yields $\mathcal{M}e_9$, then number 11 firing reaches the initial $\mathcal{M}e_0$. At this step, all the symmetrical firings are completed and the computation may deal with the asymmetrical firing from $\mathcal{M}e_5$, $\mathcal{M}e_6$ and $\mathcal{M}e_7$.

4. Properties of the Extended Symbolic Reachability Graph

In this section, the main properties of the Extended Symbolic Reachability Graph are enumerated. From a graph point of view, the inclusion of eventualities according to some symbolic markings induces the existence of implicit arcs. Therefore, we must re-define the classical notions of the paths and circuits in a ESRG, before studying the preservation of the major properties of reachable reachability graphs. In a first time, we analyze the firing sequence property and the reachability property. Then, the property on states and on transitions are considered. For sake of concision, the proves are not reported in this paper: see [12]. Moreover, similar proofs may be found in [4][11]. The properties are given, with respect to a given Well-formed Net.

4.1. Paths of an ESRG

The following property expresses that an extended symbolic path is built from the arcs of the ESRG and from the relation of inclusion between eventualities and symbolic markings.

Definition 4.1 Extended symbolic path and circuit in ESRG

Let us consider the following ordered set of arcs of the ESRG:

$\varphi=\{S_0 \to S_1 , S_1' \to S_2 , \dots , S'_{n-1} \to S_n \}$

The φ set is said to be a path of the ESRG if and only if one of the three properties hold:

$\forall S_i , S_i', i \in 1..n,$

- $S_i = S_i'$,
- S_i is an eventuality of S_i' ,
- S_i' is an eventuality of S_i .

Moreover, the φ set is a circuit if and only if the two properties hold:

- φ is a path
- $S_0 = S_n$.

<u>Notation</u>

Let t be a transition, and $\mathcal{S}$ and $\mathcal{S}'$ be two nodes of the ESRG.

$\mathcal{S} \xrightarrow{(t, \hat{c})} \mathcal{S}'$ represents a extended symbolic arc reaching $\mathcal{S}'$ from $\mathcal{S}$, labeled by $(t,\hat{c})$.

$\mathcal{S} \xrightarrow{\varphi} \mathcal{S}'$ represents an extended symbolic path, φ, reaching $\mathcal{S}'$ from $\mathcal{S}$.

$\mathcal{M}(\delta > \mathcal{M}'$ represents a sequence of standard symbolic firings, δ, reaching $\mathcal{M}'$ *from* $\mathcal{M}$.

$\mathcal{M}[(t,\hat{c}) > \mathcal{M}'$ represents a standard symbolic firing of t for the $\hat{c}$ instance, reaching $\mathcal{M}'$ *from* $\mathcal{M}$.

M $\mathcal{S}$ means that M is an ordinary marking and it is represented by the $\mathcal{S}$ node of the ESRG.

$M[(t,c) > M'$ represents an ordinary firing, reaching M' from M.

$M[\sigma > M'$ represents a sequence of ordinary firings, reaching M' from M.

$[M_0>$ is the set of reachable marking from M_0. $\mathcal{M}e_0$ is the initial extended marking.

4.2. Firing and Reachability Properties

The first property expresses that any ordinary firing sequence is represented by an extended symbolic path. The second states the relationship between extended symbolic arc and ordinary firing.

Proposition 4.1 Preservation of firing sequences

Let M and M' be two ordinary markings and
let σ be such that: $M[\sigma>M'$,

then, $\exists$: $\mathcal{S} \xrightarrow{\varphi} \mathcal{S}'$ with $M \in \mathcal{S}$ and $M' \in \mathcal{S}'$.

Proposition 4.2 *Relationship between extended symbolic arc and ordinary firing.*

Let $\mathcal{S} \xrightarrow{(t, \hat{c})} \mathcal{S}'$ be an arc of the ESRG, then:

$\forall M \in \mathcal{S}$, $\exists M' \in \mathcal{S}'$, $\exists c \in C(t)$ $|$ $M[(t,c)>M'$.

Remark

Unlike to SMG, the knowledge of an extended symbolic firing sequence in a ESRG does not allow to find the equivalent ordinary firing sequences. In fact, the ability to preserve firing sequences concerns transitions, but not their instances. This is due to our wishes of concision in the representation of ESRG and to our focusing on the preservation of the major property which is the reachability property. However, this leads us to define only sufficient conditions for more accurate properties (see property on states §4.3 and on transitions §4.4).

The following property expresses that any ordinary marking of a ESM is reachable from any marking belonging to the initial ESM.

Proposition 4.3 Reachability

Let $\mathcal{S}$ be a node of the ESRG.
$\forall M \in \mathcal{S}$, $\exists$ σ such that: $M_0[\sigma>M$.

The former properties, on reachability and firing sequence, can be summarized as follows:

Proposition 4.4 Reachability equivalence

An ordinary marking is reachable if and only if it is represented by a node of the ESRG.

4.3. Properties on States

In this section, we study "home space properties" and "dead marking" properties. Let us recall the following definitions: (1) a set of markings is said to be "home space", if and only if from any node, one of its marking is reachable; (2) a marking is said to be a dead marking, if and only if it does not have any successor (i.e. there is no enabled transition from it). The following property expresses a sufficient condition for a node to represent a home space of markings.

Proposition 4.5 Home space of markings

Let $\mathcal{S}$ be a node of the ESRG and M($\mathcal{S}$) the set of ordinary markings represented by $\mathcal{S}$.

M($\mathcal{S}$) is said to be a home space if the following path belongs to the ESRG:

$$\{\mathcal{S}' = \mathcal{S}_1 \xrightarrow{\varphi g} \mathcal{S}_m \text{ , } \mathcal{S}_m' \xrightarrow{\varphi i} \mathcal{S}_n' = \mathcal{S}\} \text{ with,}$$

- φg is a path, the arcs of which corresponds to generic symmetrical firings;
- φi is a path, any arc of which corresponds to either an instantiated symmetrical firing or an asymmetrical firing.
- Optionally, φg or φi may not exist.

Proposition 4.6 Unavoidable home space of markings

Let $\mathcal{S}$ be a node of the ESRG and M(S) the set of ordinary markings represented by S.

M(S) is said to be an unavoidable home space if the two following points hold :
- M(S) is a home space of markings ,
- S belongs to all the circuits of the ESRG.

Proposition 4.7 Dead marking (i.e. pseudo liveness)

Let M be an ordinary marking reachable from M_0.

M is said to be dead if and only if there is no output arc from the eventuality or the standard symbolic marking which represent it.

4.4. Properties on Transitions

Several definitions of liveness exist. In this section, we deal with transition and their properties of liveness and quasi-liveness. Let us recall that: a transition is quasi-live if and only if it is enabled from at least one ordinary reachable marking; a transition is "live" if and only if, from any ordinary reachable marking, there is a sequence of enabled firings containing it.

Proposition 5.7 Quasi-liveness

Let t be a transition.

t is quasi-live if there is an arc , the label of which contains t.

Proposition 5.8 Liveness

Let t be a transition.

t is live if the three following points hold: (1) t is quasi-live; (2) $\mathcal{M}_0$ is uniform; (3) $\mathcal{M}_0$ represents a home space of markings.

5. Conclusion

The technique of Extended Symbolic Reachability Graphs (ESRG) is derived from the symbolic theory, based on Well-formed Nets. By relaxing the notion of admissible permutations of objects in static subclasses, we have extended the notion of equivalence classes of objects, to take into account the asymmetrical behavior caused by some classes. Hence, symbolic markings may be partially unfolded to fire asymmetrical transitions in an instantiated way, while symmetrical transitions are fired generically. The fact that the unfolding technique is only partial and dynamically performed (only when necessary) induces that the ESRG theory allows one to build more reduce graphs than with the classical symbolic theory.

Like for standard Symbolic Reachability Graph, an algorithm which computes ESRG, automatically and efficiently, has been highlighted. Finally, we have shown that the reachability property of markings is preserved on such graph, hence, all the safety properties can be directly checked. However, the wished concise representation of ESRGs has caused that we can obtain only sufficient conditions for more accurate properties like home space and liveness properties. Our aim is now to enlarge the field of the preserved properties, keeping the same ability to deal with partially symmetrical systems.

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Appendix

Definition Well-formed Nets

A Well-formed Net is a height tuple WN=<P,T,C,W^-,W^+,ϕ,π, M_0>.
Let us recall that P is the set of places, T if the set of transitions, C={C_1,...,C_k} is the set of classes of objects, $W^-$and W^+, are respectively the backward and the forward incidence functions, featuring the input and output arcs of transitions, ϕ features the guards of transitions, π is a transition priority function, M_0 is the initial marking function.

Let C(r)=$\Pi C_{r(\alpha)}$ be the color domain of any r element of P$\cup$T. Moreover, let us recall that a multiset b, over a set A, is a function from A to $\mathbb{N}$, i.e. b$\in$ (A $\rightarrow \mathbb{N}$) and that Bag(A) denotes the set of all the multisets over A. Hence, the following definitions hold: W^- and W^+ map C(t) in Bag(C(p)), for all t of T and p of P; ϕ maps C(t) in {true,false}; π maps T in the set of integer values and M_0(p) associates with each p of P a multiset of Bag(C(p)).

Let us consider that any C_i class of objects is partitioned in static subclasses, the names of which is C_i^j if j is the j^{th} subclass of C_i. For reasons on clarity, we assume in this paper, that the considered classes are not ordered.

A colour function is defined as a linear combination of tuples of functions. Such last functions are defined on classes and are either constant functions, identity functions, diffusion functions (i.e. the codomain is all the elements of the considered class) or successor functions (for ordered classes).
The set of variables bound to a transition is the union of the variables used in the colour functions which valuates the arcs incident to the transition. The colour domain of any t transition may be defined using such notion of variables: C(t)=$\Pi_{v \ \ \mathrm{Variables(t)}}$C(v) , where C(v) C is the definition domain of the v variable and where variables(t) is the set of variables bound to t. An element of C(t) is called an instance of t.

Definition Standard symbolic instances

Let I be the set of class indexes Let t be a transition, the color domain of which is C(t) = $\Pi C_{\alpha(i)}$ (i$\in$Bag(I) is used to index the products and $C_{\alpha(i)}\in$C). Let $\mathcal{M}$ be a standard symbolic marking and $\mathcal{R}$ a symbolic representation of $\mathcal{M}$.
We say that ($\Pi Z_{\alpha(i)}{}^{\lambda(i),\mu(i)}$) is a symbolic instance for t wrt. $\mathcal{R}$ if and only if the following points hold :

- $\alpha(i)\leq k$, is an index of class.
- $\lambda(i)\leq\mathcal{R}$.m($\alpha$(i)) , is an index of dynamic subclass, wrt. $C_{\alpha(i)}$.
- $\mu(i)\leq\mathcal{R}$.card($Z_{\alpha(i)}{}^{\lambda(i)}$) , is a number of object, wrt. $Z_{\alpha(i)}{}^{\lambda(i)}$.

Thus, a symbolic instance for a transition is a product of dynamic subclasses. A dynamic subclass may occur several times: if some μ values are equal, with respect to the same dynamic subclasses, then the same object is referred . In practise, one can note that (λ) and (μ) are precised only if necessary.